Stochastic Utility Maximising Dynamic Programming Applied to Medium-Term Reservoir Management

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To Su and Liam

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Abstract

Medium-term reservoir management is a classic planning problem to which stochastic dynamic programming has been applied. An aspect of reservoir management modelling often neglected is 'risk', although it has been identified as being of prime importance. A utility function can imply an attitude to risk, and in this thesis, a modified stochastic dynamic programming model (SUMDP) is presented which can maximise expected utility, where utility is defined over the range of terminal storage and 'wealth' outcomes and hence is dependent on all decisions made over the planning horizon.

SUMDP is applied to reservoir management in regulated and deregulated representations of the New Zealand electricity system. Experimental results showed that increasing the relative risk aversion to low terminal wealth values reduced the mean and variability of wealth and was achieved by conserving water and hence increasing storage. This effect was amplified by the contract level of the hydro firm in a deregulated case where the reservoir firm was a price setter with financial contracts and the remaining players were price takers.

SUMDP can be applied to other problem classes, one of which is stochastic route choice in acyclic networks. SUMDP is discussed in this context and applied to some example problems. Rather than a single (static) route choice decision being optimal at each node of the network, SUMDP produces optimal non-static decisions which are dependent on the accumulated time taken to reach the node and take into account the utility associated with the time taken to travel the route. There are few approaches discussed in the literature which produce non-static solutions, consider uncertainty, and consider risk, so SUMDP also contributes to this literature.

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Chapter 1

Introduction

Decision making is an activity that permeates our lives in numerous situations on a daily basis. The context and importance of these 'decision situations' (also referred to as decision problems) is wide and varied. Many of these situations involve the same type of decision process: choosing from a set of actions and considering the possible consequence(s) of this decision. While easy to describe, the effort and time required to select an action can range enormously. The technique(s) used to select from alternative actions can be as wide and varied as the problem at hand, ranging from unconscious reactions, to simple rule of thumb and/or 'gut feeling' approaches, to large and complex mathematical optimisation models. For complex systems and/or decision situations, mathematical models can be a useful, if not necessary, tool for decision making because they incorporate aspects of the system and problem which can not be factored in to the simpler approaches.

So what is a mathematical optimisation model? Essentially, it is a mathematical representation, or abstraction, of a decision problem faced by a DM. Let Q denote the set of prescribed feasible decisions available to the DM. A given decision, q, is constrained to be a member of Q, so $q \in Q$. Central to optimisation models is the notion of an objective function, which reflects the value to the DM of the consequences of decision(s), and is used to discriminate between alternative decisions. Let this objective function be referred to as $f(q)$. In order to find the 'best', or optimal, value of q, the DM solves the following optimisation problem: maximise $f(q_1,...,q_n)$ subject to $q \in Q$. This simplified model can be extended and modified in a number of ways, depending on the problem being considered.

Stochastic sequential (or multi-period) decision problems are one type of problem which can be approached using mathematical optimisation. These problems incorporate uncertainty about internal and/or external parameters; they are sequential because they involve making (often similar) decisions repeatedly, and often over time. Examples of these problems can be found in the areas of inventory management, transportation, and finance. Determining the optimal value(s) of q is difficult for these problems because the consequences of alternative actions must usually be compared at different points in time, and uncertainty can affect what actions are feasible at different points in time, as well as the consequences themselves. An important characteristic of many of these problems is that uncertainty is resolved during the planning process, and the DM has the ability to make decisions which are conditional on the history of outcomes. A suitable solution technique for these problems will therefore produce a dynamic solution rather than a static solution, because the latter does not allow for any adjustment through the planning horizon.

Reservoir management, uncertainty, and risk

Reservoir management is an example of a stochastic sequential decision problem. In essence, the problem involves determining reservoir releases over a specified time frame in order to maximise the 'value' to the DM of the consequences of releasing or storing water. In the context of reservoir operation in an electricity system, which is the focus of this thesis, the 'consequence', or return, from a release is the cost or profit from the electricity generated from that release. It is an interesting and complex problem because water is a storable commodity, so there is a continuous process of deciding whether to release it now or to store it and release it later. This trade-off is made on the basis of uncertainty about future physical and economic factors (e.g., inflows and prices), which is often referred to as risk.

The importance of considering uncertainty and/or 'risk' can be found throughout the reservoir management literature. Yeh (1985) stated that "... *the trade-off between return and the associated risk is of prime importance...*" (p. 1805). Reznicek and Cheng (1991) comment that "... *long-term reservoir operation has to trade off benefits and risks associated with achieving the benefits...*" (p. 241). Larsen and Bunn (1999) state that "...*Governments are risk averse when it comes to the security of the electricity system...*" (p. 339). Philbrick and Kitanidis (1999) state that "...*operators are typically risk averse*" (p. 136). For electricity systems dependent on hydro electricity and/or where uncertainty is prominent, the implication is clear. This was recognised by Read et. al. (1992) in their commentary on the use of operations research models in the New Zealand energy sector. They stated that "... *a major deficiency in our modelling and planning was the inadequate attention paid to risk, and the flexibility of different solutions to cope with it*" (p. 244).

Systematic procedures for analysing risk are desirable in an organisational setting (Ward, 1997), though the relationship between DM concepts of risk and theoretical concepts is a subject of continued research and debate (March and Shapira, 1987). But what is risk? The concept of risk can take on a number of definitions depending on the situation at hand. Risk is usually referred to in conjunction with, and sometimes in place of, terms such as variation, utility, preferences, uncertainty, and hazards. The concept of risk adopted here is akin to that in decision analysis, which is a prescriptive framework for choosing among a set of pre-specified alternatives with uncertain outcomes. What is important is not risk, per se, but the DM's preferences towards the consequences of decisions. A utility function can be used as a mechanism for reflecting these preferences, and investigating trade-offs between preferences towards different consequences (Keeney and Raiffa, 1976). Attitudes to risk, such as the DM being risk averse or risk seeking, can be derived from the form of a utility function. With a utility function defined over the set of consequences, the decision problem becomes one of maximising expected utility as opposed to maximising expected value.

In reservoir management, risk is generally associated with uncertainty about future physical and economic factors, and their impact on system operation and costs. Reservoir management models that incorporate stochasticity into the optimisation are therefore often deemed to consider 'risk'. But in general, stochastic models have an objective which maximises the expected value of the returns from release plus the value of storage held at the end of the horizon. (The latter term is required to reflect the value of storing water for use after the planning horizon). Maximising the expected value of these returns, though, does not explicitly consider 'risk' as implied by a utility function. Furthermore, if a utility function (defined over 'end of horizon' performance) is used to assess the performance in each period, the resulting policy will not necessarily correspond to that which maximises utility of the performance at the end of the planning horizon (Bard and Bennett, 1991).

There exists extensive literature on utility functions and decision making. Research has addressed aspects such as the form and attributes of utility functions, the suitability of utility functions for prescriptive and descriptive decision making, and temporal issues. Research relevant to this work will be discussed in a later section. It is worth mentioning here, though, that decision analysis can be difficult to apply to strategic decision problems (Keeney and Raiffa, 1976). The nature of uncertainty and the set of available decisions can be difficult to prescribe (e.g., the location of an airport or factory, the price of a good, or a nation's tax structure). Moreover, there exist multiple consequences which are difficult, if not impossible, to subjectively or objectively estimate or trade off against each other. On the other hand, tactical (or operational) decision problems are often well defined, both in terms of the decisions available, and the consequences of those decisions. For this reason, a utility function defined over these consequences is a more viable proposition, as is a systematic analysis of the relationship between decisions and utility.

Medium-term reservoir management is a relatively well defined tactical decision problem because the same type of decisions are made repeatedly. In simple terms, the decision in each period is how much to release, which has some immediate return (e.g., profit or cost) and, in conjunction with inflow uncertainty, influences the storage in the next period. If the problem were represented using a decision tree, the nodes would represent points in time and storage levels at which a decision can be made (assuming hydrological and economic events were not correlated). Chance nodes would denote uncertain inflows and alternative decisions at the node would correspond to alternative feasible release levels. The consequence at the end of branch (a sequence of releases, inflows, and storage levels) of the decision tree would be the accumulated returns and storage level (referred to as attributes). In decision analysis, these two attributes "*...characterise the full cognitive impact of that position point in time and space*" (Keeney and Raiffa, 1976 p. 31). This is a reasonable assumption for medium-term reservoir management problems where the objective has been defined as maximising the expected value of accumulated returns and storage at the end of the planning horizon. Therefore, the possible combinations of these two attributes describe a domain over which a utility function could be defined. The objective becomes one of maximising the expected *utility* associated with end of horizon accumulated returns and storage.

Maximising expected utility of accumulated returns and storage appears to be a potential way of representing the DM's attitude to risk, via a utility function. The reason for this is that the expected value of utility is a function of all the decisions and realisations of uncertainty which occur during the planning horizon. A major complication with an objective that maximises utility in a sequential decision problem, though, is that it is (generally) non-separable. Techniques for solving reservoir management problems with uncertainty (and other stochastic sequential decision problems) include simulation, stochastic programming, and stochastic dynamic programming (SDP). SDP can handle the dynamic decision structure of many time periods, temporal resolution of uncertainty, and non-linearities in the return and objective function. However, conventional SDP, along with other techniques, relies on the objective function being temporally separable. This can limit the type of situations able to be modelled, including one with a utility maximising objective as defined here. Decision analysis, while able to handle the objective of utility maximisation, requires enumeration of all branches of a decision tree. Each branch corresponds to a scenario that could occur. For sequential decision problems with multiple stages (periods), states (storage levels) and decisions (release), the number of branches of the decision quickly becomes impossible to analyse.

It would be desirable, then, to devise a method of handling the non-separable objective while maintaining the computational advantages that result from the stage-wise decomposition as well as the other modelling advantages that SDP offers (Yeh, 1985). One way to address the separability issue is to increase the number of states, although this increases the computational requirements, and in the worst case, results in the problem being solved using complete enumeration. However, Kaye and Read (1998) observed that a utility maximising objective defined over the accumulated returns only required the addition of a single state variable. The computational effort required is therefore increased, but not necessarily prohibitively. They describe the application of the technique to a centralised 'cost minimising' reservoir management problem. Ranatunga (1995) applied this approach to the problem of short-term scheduling for a thermal power station facing price uncertainty. More recently, Craddock et. al. (1999) applied it to a deregulated reservoir management problem which they solved using a Dual SDP technique developed from that described in this thesis.

This thesis studies Kaye and Read's technique applied, in the first instance, to medium-term reservoir management. The approach – referred to as **S**tochastic **U**tility **M**aximising **D**ynamic **P**rogramming (**SUMDP**) – is illustrated using an aggregated representation of the New Zealand electricity system, a system which is highly dependent on hydro electricity. Indeed, the situation in New Zealand in 2001 has highlighted the impact of inflow uncertainty on reservoir operation. After a dry summer and inflows consistently below average, storage levels are such that consumers are being encouraged to reduce electricity consumption so as to avoid black outs, and electricity prices have reached historically high levels. In this situation, the implications of the attitudes of hydro companies, and the New Zealand Government, to inflow uncertainty and storage levels is receiving considerable scrutiny.

After discussing reservoir management and introducing SUMDP, a regulated case is considered, where the problem is to schedule reservoir release (generation) and thermal plant so as to meet a fixed deterministic demand. The objective is to maximise the utility of the accumulated cost and ending storage. A deregulated case is then considered, where the reservoir, while contracted for a fixed quantity in each week, also trades with the spot market, where it operates as a price setter. Thermal plant are treated as price takers, and the objective is to maximise the combined utility of revenue from release and ending storage. In both cases, the models are used to investigate the differences in system performance resulting from using utility functions which imply different preferences for accumulated return and storage. The results are compared to those derived using a conventional objective of maximising the expected cost or profit.

SUMDP applied to medium-term reservoir management receives the majority of the attention in this study. However, SUMDP can also be applied to another class of sequential decision problems known as stochastic route choice (SRC) problems. A general SRC problem involves finding a route (sequence of arcs) through a directed acyclic network which minimises the sum of the expected arc costs, which are uncertain. There are a relatively small number of studies which address the case where the DM's preference towards the total cost is reflected by a utility function. Few techniques produce a dynamic solution, and these do not consider utility functions. SUMDP, while relatively inefficient, can produce a solution which is dynamic *and* utility maximising. SUMDP applied to SRC problems is therefore briefly explored.

Thesis outline

To summarise, while risk has been identified as an important aspect of reservoir management, few stochastic models have explicitly incorporated risk. This thesis presents a variant of stochastic dynamic programming that explicitly considers risk (insofar as it is implied by a utility function) and applies it to medium-term reservoir management using the New Zealand electricity system. The approach is also studied for stochastic route choice problems where utility maximisation has received considerable attention but few techniques have been published that produce dynamic utility maximising solutions with uncertainty.

The layout of this thesis is as follows:

Chapter 1: Introduction.

Chapter 2: Modelling approaches to reservoir management are reviewed, with an emphasis on risk and competitive market issues.

Chapter 3: Issues involved with combining utility with sequential decision problems are discussed. SUMDP is introduced and related approaches are reviewed.

Chapter 4: The application of SUMDP to classical 'regulated' medium-term reservoir management using a representation of the New Zealand electricity system is discussed.

Chapter 5: Implementation issues are discussed, including algorithmic modifications which reduce the computational requirements of the solution approach.

Chapter 6: Optimisation and simulation results are discussed for the regulated case. The impact on the release schedules and system performance from varying the curvature and positioning of the utility function is discussed.

Chapter 7: SUMDP is applied to deregulated medium-term reservoir management using a representation of the New Zealand electricity system. The benefit function is modified to incorporate a fixed contract quantity.

Chapter 8: Optimisation and simulation results are discussed for the deregulated case. The impact on the release schedules and system performance from varying the curvature of the utility function, as well as the contract level, are discussed.

Chapter 9: Some extensions to SUMDP for reservoir management are presented. These include the simultaneous management of thermal plant, and a Stackelberg gaming model, both of which effect the derivation of the return in each period.

Chapter 10: SUMDP is applied to stochastic route choice problems. The relevant literature is reviewed, and SUMDP is illustrated using some simple example problems.

Chapter 11: Conclusions.

Chapter 2

Reservoir Management

2.1 Introduction

Stochastic dynamic programming (SDP) is a useful technique for reservoir management problems because the non-linear and stochastic aspects of the problem can be incorporated into the problem formulation (Yeh 1985, Reznicek and Cheng 1991). The planning horizon divides naturally into discrete time periods (the stages) with storage as the state variable which links the stages. For each stage and state, a release is determined which maximises the expected return (or minimises the cost) of the decision at each stage and the value of storage at the end of the horizon; future uncertainty is explicitly factored into the release decision.

SDP is introduced as an approach to reservoir management in Section 2.2. Reservoir management for the New Zealand system is discussed in Section 2.3. Sections 2.4 and 2.5 discuss how 'risk' and competitive electricity market issues have been incorporated into reservoir management models.

2.2 Modelling approach

For many years, a common starting point for research in the area of reservoir optimisation has been the state-of-the-art review of reservoir management models by Yeh (1985). Models were grouped by the type of optimisation technique, with those groups being primal and dual variants of linear programming, non-linear programming, and dynamic programming. Approaches can also be classified by the number of reservoirs considered, the length of the planning horizon, the nature of the objective, return, transition functions, and the nature of inflows. Deterministic methods are rarely suitable because the characteristics of reservoir management problems do not satisfy the assumptions required to produce optimal release schedules (Philbrick and Kitanidis, 1999). Most techniques that explicitly consider uncertainty, rather than using heuristic means, are either a variant of stochastic linear programming (SLP) or stochastic dynamic programming (Philbrick and Kitanidis, 1999). Here, we are concerned with a SDP approach to reservoir management.

Due to the nature of the discussion in the remainder of the chapter, it is worth introducing a classical (and typical) SDP approach, as in Yeh (1985) for example. For a T-stage problem, the release schedule (which is a function of the storage level in t) which maximises the sum of the returns in each period can be found by solving

$$f^t(s^t) = \max_{q^t} \sum_{a^t} \Pr(a^t)(r^t(q^t) + f^{t+1}(s^{t+1}))$$

subject to $$s^{t+1} = s^t - q^t + a^t$$

$$q^t \in Q^t(s^t),\ s^t \in S^t \tag{2.1}$$

for $t = T-1, \ldots, 1$ and given $f^{T+1}(s^{T+1})$ defined over all ending storage levels. The state of the system in each period is the storage level $s^t \in S^t$, the decision variable in each period is the release $q^t \in Q^t(s^t)$. The uncertain and discrete inflow in t is a^t which occurs with probability $\Pr(a^t)$. The return, or benefit, from release q^t is reflected by $r^t(q^t)$. Satisfaction of monotonicity and separability conditions (Nemhauser, 1966) is sufficient for this decomposition. When the returns are additive (and in this case independent), a stage-wise decomposition is possible because these conditions are satisfied. This approach is discussed in more detail in Chapter 3.

A drawback with SDP (and SLP) is that it has limited application to systems which require multiple state variables. This can occur under a variety of circumstances e.g., multiple reservoirs, correlated inflows, and when modelling other aspects of the problem. In a SDP context, approaches have been proposed for reducing the impact of increasing the state space by reformulating the problem (Read, 1989; Archibald et. al., 1997) and reducing the computational effort required to approximate the future value function (e.g., Johnson et. al., 1993; Chen et. al., 1999).

2.3 Reservoir management in New Zealand

Managing the provision of hydro electricity is a relatively important issue for many countries around the world e.g., Argentina, Australia, Brazil, Chile, New Zealand, and Norway. The reliance on hydro generation varies, though, depending on factors such as the storage capacity of reservoirs, the availability of alternative generation sources, and network configurations. From a modelling perspective, the important characteristics of the New Zealand system are the reliance on hydro generation, the spatial dispersion of generation plant, and the stochastic nature of inflows (Scott, 1997). Hydro generation accounts for approximately 70% of annual national electricity demand. Because the aggregate storage capacity is only 6 weeks (approximately) of national demand, and thermal generation is relatively expensive and limited in capacity, detailed planning of reservoir operation is crucial. In addition to natural inflows being highly uncertain, New Zealand is geographically isolated from any other electricity system, so can not rely on external electricity sources if required. A large proportion of hydro electricity is provided from two reservoir systems; for medium and long-term planning these are often aggregated into a single reservoir. These reservoir systems are located in the South and North Islands, with a DC-link used for transferring power between the two islands.

Several reservoir management models have been developed/implemented and/or tested using the New Zealand system. For example, Read's (1989) two reservoir Dual SDP model was the basis of the model used for reservoir management by the Electricity Corporation of New Zealand (ECNZ). Yang and Read (1999) extended Read's model by considering correlated inflows. Scott (1997) extended Read's model by considering operation of the aggregated reservoir in a deregulated electricity market, with weekly behaviour represented using a Cournot model. Recently, Craddock et. al. (1999) have extended Read's and Scott's model by augmenting the state space of their two-reservoir dual SDP model with an auxiliary 'wealth' variable, utilising the approach first developed in this thesis.

2.4 Reservoir management models incorporating risk

Risk has generally been incorporated into constraints rather than the objective (Yeh, 1985) and the models are only partially stochastic (Philbrick and Kitanidis, 1999). Chance constrained models, also referred to as reliability constrained models, solve a model which has an additional set of constraints which have a probabilistic term. These constraints can be added to LP and DP models. Askew (1974a) described one of the first approaches to handling risk in the context of reservoir management. He described a 50 period (years) planning model for a single reservoir with stochastic inflows with independent distributions. Each period had an associated target release level, which corresponded to demand. The risk was defined in terms of a 'failure', where a failure corresponded to there being insufficient storage to meet demand. If a failure occurred, an economic penalty w was incurred, otherwise there was no penalty. This penalty was assumed constant, so the magnitude of the penalty did not reflect the magnitude of the shortage. The recursive relation was defined as

$$f^t\left(S^t\right) = \max_{q_t} \sum_{a^t} \Pr\left(a^t\right)\left(r^t\left(q^t\right) + \alpha^t f^{t+1}\left(S^{t+1}\right) - w\right) \tag{2.2}$$

where α^t is the discount rate applied to the future value function. Policies were found for combinations of reservoir size, inflow distribution, benefit function, and discount rate. The impact of w on the policies, expected net benefit, and frequency of failures was compared. As w was increased, the expected benefit decreased, as did the frequency of failures. Details regarding the size of the failures were not supplied. However, the 'high w' policies usually involved lower release levels, so the magnitude of the shortages presumably decreased also.

Using the same recursive relation described in Askew (1974a), Askew (1974b) describes a DP model capable of handling chance constraints. Chance constraints were defined as a constraint on the probability of a variable attaining a value outside of some predefined bounds. As in Askew (1974a), the reliability constraint was not explicitly considered when determining the optimal release policy. Instead, the values of the constraint variables were measured during the optimisation and the feasibility evaluated post optimisation. In Askew (1975), w was defined differently than in Askew's earlier work; rather than being an explicit cost as in the earlier work, w was added to the discount rate (d) which was applied to the value function. The value of w was described as a form of risk premium in that immediate benefits were valued more highly than future benefits, depending on the value of $d+w$. The purpose of w remained the same in all three papers, being used solely as an incentive to produce a schedule which was then able to be evaluated with regard to any reliability constraint(s). Therefore, the resulting schedule may not be optimal to the original problem, and this point is noted by Askew (1974b).

Because the penalty is included in the calculation of the weekly benefit, the overall objective remains that of maximising an expected value. Therefore, the objective remains separable and additive, and the manager is implicitly risk neutral towards the distribution of end-of-horizon benefits (inclusive of the penalty). The reliability constraint(s) must also be additive over time (Rossman, 1977). In the SUMDP model described later in this thesis, a low release level can result in a shortage cost. This shortage cost is similar to Askew's w penalty, though it is dependent on the magnitude of the shortage. SUMDP is different, though, because the DM's utility of total cost (inclusive of the shortage cost) is considered when determining the optimal release in a given period.

In their comment on Askew (1974b, 1975), Sniedovich and Davis (1975) briefly discuss how reliability constraints, which are a function of the values of system variables over the entire planning horizon, can be incorporated directly into the DP algorithm. They describe (briefly) how the state space can be augmented with the reliability variables, and these can then be constrained as described in Askew (1974b). For example, Askew describes the following reliability constraint: the average number of occasions during the life of the project on which it will be unable to supply its forecast release must not exceed a given maximum (γ). Sniedovich and Davis suggest that this constraint can be handled by defining y^t as the expected number of failures up to the beginning of period t. The constrained optimum could then be found by maximising the expected value of benefits, while satisfying constraint $y^t \leq \gamma \quad \forall t$, where y^T=0 (here T is the last period). The definition of y^t is similar to the definition of the accumulated wealth state variable (w^t) considered in this thesis. However, Sniedovich and Davis do not consider the issue of the manager having preferences towards the outcome of that variable.

Rossman (1977) also critiques Askew's approach and develops a reliability-constrained DP approach from a Lagrangian perspective. He models the reliability constraint defined in Askew i.e., a constraint on the expected number of failures. The recursive relation is of a similar form to Askew's because it too includes a penalty for violating the reliability constraint (Yeh, 1985). He also defines a number of other reliability constraints, some of which are not possible to incorporate into Askew's CCDP approach because they are not separable.

The chance-constrained models described above do not take into account the storage level at the end of the horizon, nor is it mentioned by any of the authors. This is a tolerable assumption if the problem is framed in terms of the entire life of a reservoir and a stationary policy is desired. If that were the case, though, it would be preferable to constrain storage in the last period to be zero or some other desired storage level e.g., for future recreation and/or environmental use. One way to reduce the incidence of these outcomes, and hence accommodate risk in some way, is to solve a model with chance (or reliability) constraints on the probability or frequency of an outcome occurring. A drawback is that the preferences are assumed to be linear for outcomes that are feasible with respect to the constraint. Chance-constrained models have also been developed for multiple reservoir and/or multiple objective problems (Reznicek and Cheng, 1991).

A technique related to chance-constrained programming is reliability programming. Reliability programming uses risk loss functions to penalise the risk of choosing an infeasible solution (Reznicek and Cheng, 1991). Reznicek and Cheng (1991) present an overview of stochastic linear programming models which include the reliability programming constraints and objective terms. These all involve maximising the sum of the benefits less any penalties for constraint violations. The approach presented by Reznicek and Cheng is considered a multiobjective approach because two loss functions are considered. However, the reliability constraints are still defined such that a separable objective(s) can be defined.

A recent approach which does not use a utility function is the portfolio management model of Fleten et. al. (1999). The objective is to maximise the expected operating profit (spot and contract revenues/costs) plus end-of-horizon reservoir values:

$$\max \left\{ \mathrm{E}\left[\sum_{j \in J} \left(I_j - Pen\left(I_j, \hat{I}_j\right) + V\left(S_T\right) \right) \mid p, v \right] \right\} \tag{2.3}$$

At the end of each profit period (*j*), expected profit I_j is penalised, should it fall below some predefined target level ($\hat{I}_j$). Variables p, v denote correlated price and inflow uncertainty, $Pen\left(I_j, \hat{I}_j\right)$ is the penalty function, with $Pen\left(I_j, \hat{I}_j\right)>0$ for $I_j < \hat{I}_j$, and $V\left(S_T\right)$ is the value of end-of-horizon storage. They argue that these (convex) penalty functions form implicit (concave) utility functions because the marginal value of increasing income is higher when below the target. This may satisfy the economic definition of a decreasing utility function, but does not imply anything about the DM's preferences for different outcomes.

Essentially, these penalty functions are a variant of the single-period penalty functions of chance constrained programming. The difference is that the penalty is defined over the performance in multiple periods and requires an augmentation of the state space as in SUMDP. Performance and penalties are measured in like terms, though, so their approach is specific to the portfolio management context within which it was described. Justifying penalty functions for problems with non-financial objectives or non-comparable financial objectives would be difficult. More importantly, though, these methods do not deal with risk as it might be implied by a utility function.

2.5 Market issues

Traditionally, the electricity industry has been managed by Government-owned or regulated entities. In the last decade, though, it has been restructured and deregulated to various degrees throughout the world. This deregulation has encompassed the supply, transmission, and delivery of electricity. In New Zealand, deregulation of electricity supply sector has resulted in the creation of a number of smaller companies that operate in a commercial environment. A consequence of this is that instead of a single centralised firm minimising cost, there are now several firms maximising profit. The objectives and purposes of the planning models used by these firms must therefore be adapted, and reservoir management models are no exception.

While the dynamics of reservoir systems are well understood (and modelled), the dynamics of electricity markets are a newly emerging modelling area. In addition to the complexities of physically generating and transmitting electricity, and uncertainty about environmental parameters, which affect all systems, additional complexities and issues such as the actions and interactions of multiple firms, risk, contracts, and the management of the physical and financial assets of firms over time must be considered. There are numerous publications which address various combinations of these factors. However, there are very few models that consider reservoir management in a competitive wholesale electricity market (Scott, 1997).

One of the issues arising from deregulation of electricity and other utility sectors is the influence that firms have on the price in the market, and hence the potential market power of firms. The general area of market power and associated strategic behaviour has been a subject of recent modelling effort, though there are few applications with detailed hydro modelling. One model used for representing the strategic behaviour of firms is the Cournot-Nash model. Cournot competition assumes that firms compete on the basis of quantity, and each firm makes an offer based on an assumption of the output of all other firms. A Nash equilibrium is found when there is no incentive for any of the competing firms to alter their generation levels. For example, Borenstein, Bushnell and Knittel (1999) present a recent critique of market power models and illustrate results of a comparison between perfect competition and the Cournot equilibrium. An alternative to Cournot competition is Bertrand competition and involves competition on the basis of price (rather than quantity). In Bertrand competition, any firm can undercut all other competitors by offering generation at a lower price, though they may not be able to physically supply the resulting demand due to capacity constraints. This in turn causes instability in attaining an equilibrium (Borenstein et al, 1999). For further discussion of issues involved with equilibrium models and electricity markets, see for example Smeers (1997).

There are relatively few published approaches for handling reservoir management in a competitive (or 'deregulated') environment where there are one or more firms with market power (Bushnell, 1998). Scott (1997) and Craddock et. al. (1999) describe medium-term reservoir management models where the firms game in a Cournot fashion. Optimal reservoir releases are determined using Dual Dynamic Programming with the state space in each period being the marginal value of storage for Scott's model, and the ratio of the marginal utility of wealth to storage in Craddock et al.'s model. The state space is discretised and each marginal value is used as the marginal cost of hydro generation when determining the Cournot equilibrium. Bushnell (1998) described a multi-period multi-firm model with Cournot competition, though didn't account for uncertainty. Hydro firms were modelled by constraining their total generation during the planning horizon (1 month), yet storage and inflows were not explicitly modelled. More importantly, these firms were implicitly assumed to be risk neutral with respect to wealth because the objective was to maximise expected profit.

2.6 Conclusions

While there are many approaches to reservoir management, there are few published approaches which systematically account for attitudes to 'risk', or at least anything other than non-risk neutral attitudes. To the author's knowledge there are no published techniques for reservoir management when a utility function is used to represent a DM's attitude to outcomes at the end of the horizon (excepting Craddock et. al. (1999) which was developed in conjunction with this research). There are also few published approaches for handling reservoir management in a competitive or deregulated, environment where there are a few firms with market power (Scott, 1997; Bushnell, 1998). In Chapter 3, SUMDP is introduced as a modified version of a conventional SDP approach to the reservoir management problem. Chapters 4-9 discuss the application of SUMDP to reservoir management in regulated and deregulated electricity markets.

Chapter 3

Utility and Stochastic Sequential Decision Problems

3.1 Introduction

SUMDP integrates aspects of decision analysis (see for example Keeney and Raiffa (1976)) with dynamic programming (see for example Bellman and Dreyfus (1962)). A classical SDP approach to reservoir management is developed in Section 3.2. Issues involved with integrating utility with stochastic sequential decision problems are discussed in Section 3.3. SUMDP is presented in Section 3.4 as an approach to handling utility in the context of stochastic sequential decision problems. Related approaches are reviewed in Section 3.5 and Section 3.5 presents conclusions.

3.2 Stochastic dynamic programming

A general stochastic sequential decision problem will now be introduced, such as might be applied in the context of classical reservoir management (e.g. Yeh, 1985). A DM controls a system over a finite planning horizon. The planning horizon is divided into t=1...T discrete stages, or periods. At any stage, the system is in one of a finite number of states, s^t, where $s^t \in S^t$. At each stage and state, the DM can make a decision, q^t, which is one of a prescribed finite number of possible actions $q^t \in Q^t$. Uncertainty in the period is represented by a^t. For reservoir management, s^t is the storage level at the beginning of *t*, q^t is the release made in *t*, and a^t might be inflow and/or price uncertainty.

The state of the system evolves through time, depending on the values s^t, q^t, and a^t according to a transition function:

$$s^{t+1} = T_s^t\left(s^t, q^t, a^t\right) \tag{3.1}$$

For reservoir management, a simple form of this transition is $s^{t+1} = s^t - q^t + a^t$, where a^t is the uncertain inflow in the period. Additional factors incorporated into this transition function could include evaporation and spill.

Associated with decision q^t is an immediate consequence (or return or reward), $r^t\left(s^t, q^t, a^t\right)$, which can also be uncertain. Typically, $r^t(\bullet)$ is a monetary cost or profit which can be discounted if required. An additional factors which might be included is head effects. The actual definition of $r^t(\bullet)$ is dependent on the situation being considered. Examples include the cost of meeting demand, profit from release, power output, and the difference from a target output level. The state, decision, and uncertainty variables are assumed here to be unidimensional, but can be multidimensional.

The objective for this problem is to maximise the expected return accumulated over the stages of the planning horizon, plus the value of the state of the system at the end of the planning horizon. The problem can be stated mathematically as

P1 $$f^1\left(s^1\right) = \max_{q^1,\ldots,q^T}\left\{ \mathop{\mathrm{E}}_{a^1,\ldots,a^T}\left[\sum_{t=1}^{T} r^t\left(s^t, q^t, a^t\right) + f^{T+1}\left(s^{T+1}\right)\right]\right\}$$

subject to $$s^{t+1} = T_s^t\left(s^t, q^t, a^t\right)$$

$$q^t \in Q^t\left(s^t\right),\ s^t \in S^t \tag{3.2}$$

where $f^1(s^1)$ is the expected return given optimal operation in all periods and situations. Function $f^{T+1}(s^{T+1})$, measured in the same units as the return function, reflects the value to the DM of the state of the system at the end of the planning horizon (and at the beginning of period T+1). In effect, the problem involves not only trading off the returns between periods 1,...,T, but also trading off those returns expected to be earned during the planning horizon against those earned after the planning horizon, which are reflected by $f^{T+1}(s^{T+1})$. (There are alternatives to using a non-zero $f^{T+1}(s^{T+1})$, such as constraining s^{T+1} or penalising deviations from a target level). The optimal decision in each period is a function of the state of the system in each period, so the optimal policy has the form $\pi = \{q^1(s^1),...,q^T(s^T)\}$. Because the state of the system is uncertain for each period, there is no single state-independent policy which maximises $f^1(s^1)$; determining the optimal decisions in a given t requires that all possible futures be considered, explicitly or implicitly.

One solution technique would be to completely enumerate all possible decision sequences. For small problems this can be tractable, but it is easy to see that the number of possible futures (or scenarios) grows exponentially with the number of stages and states, rendering complete enumeration a near impossible task. An alternative is to solve **P1** using stochastic dynamic programming (e.g., Bellman and Dreyfus, 1962), which restates the above problem as T sub-problems of the form

SDP1
$$f^t(s^t) = \max_{q^t}\left\{\mathop{\mathrm{E}}_{a^t}\left[r^t(s^t, q^t, a^t) + f^{t+1}(s^{t+1})\right]\right\}$$

subject to
$$s^{t+1} = T_s^t(s^t, q^t, a^t)$$

$$q^t \in Q^t(s^t),\ s^t \in S^t \tag{3.3}$$

which are solved recursively for t= *T-1, ..., 1* (a process known as backwards recursion). The function $f^t(s^t)$ is the expected cost obtained if the system is operated optimally for all sequences of random variables from t to T. Underlying this decomposition is Bellman's 'principle of optimality', which states that "*an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting* *from the first decision.*" There are more general (and hence less restrictive) versions (e.g., Denardo, 1982; Carraway et. al. 1989).

For the solution to each of the T sub-problems to be optimal for the original problem, **P1** must satisfy separability and monotonicity conditions (Nemhauser, 1966). The separability condition requires that

$$g\left[r^1(\bullet),\ldots,r^T(\bullet)\right]=g_1\left[r^1(\bullet),g_2\left(r^2(\bullet),\ldots,r^T(\bullet)\right)\right] \tag{3.4}$$

for real valued functions g_1 and g_2. The monotonicity condition requires that g_1 is a monotonically non-decreasing function of g_2 for every decision in $t=1$ (e.g., for every q^1 in **P1**). Decomposition is then possible because

$$\max_{q^1,\ldots,q^T}\left\{g\left[r^1(\bullet),\ldots,r^T(\bullet)\right]\right\}=\max_{q^1}\left\{g_1\left[r^1(\bullet)\right],\max_{q^2,\ldots,q^T}\left\{g_2\left[r^2(\bullet),\ldots,r^T(\bullet)\right]\right\}\right\} \tag{3.5}$$

For the proofs underlying these conditions, see for example Nemhauser (1966) or Kall and Wallace (1994). Note that these conditions apply to a case with stochastic returns when the uncertainty variable is independently distributed.

The size of the problem is now linear in the number of stages. However, in order to approximate $f^t\left(s^t\right)$, the state and decision variables are often discretised into a finite grid of points or levels. This is sometimes referred to as discrete stochastic dynamic programming, though SDP is used herein. The value of $f^t\left(s^t\right)$ is calculated for the discrete values of s^t by solving **SDP1**. This involves evaluating $f^{t+1}\left(s^{t+1}\right)$ for feasible q^t. Because $f^{t+1}\left(s^{t+1}\right)$ is only known for the discrete values of s^{t+1}, $f^{t+1}\left(s^{t+1}\right)$ must be estimated for s^{t+1} lying off the grid. Linear interpolation is the most often used technique (Johnson et al, 1993). If there are n state variables each discretised at x grid points then $f^{t+1}\left(s^{t+1}\right)$ must be evaluated wx^n times at each stage, where q^t is discretised at w points.

Alternative methods for approximating $f^{t+1}\left(s^{t+1}\right)$ have been proposed (see for example Johnson et al (1993) or Chen et al (1999)). Some of these have given rise to variants of traditional 'primal' stochastic dynamic programming (e.g., Read (1989)). Due to SDP's flexibility and the multi-state nature of problems, techniques that reduce the computational requirements of determining $f^t(s^t)$, whether by approximation or restructuring the problem, are ongoing research areas.

3.3 Utility and stochastic sequential decision problems

The importance of considering risk in reservoir management has been discussed in the previous chapters. It was pointed out earlier that risk is an important aspect of reservoir management, yet there are few published approaches for incorporating risk. In the previous section, a classical SDP approach to reservoir management (**SDP1**) was presented as an approach to solving **P1**. The objective of **P1** (and **SDP1**) is to maximise the expected benefits from release plus the value of storage held at the end of the horizon. As a result, the DM is implicitly risk neutral; decisions are only compared on the basis of the expected value of the consequences. A utility function can be used to represent DM preferences towards consequences, and depending on the shape of that utility function, will reflect the DM's risk attitudes.

The general form of the utility function to be considered is

$$U\left(r^1(\bullet),\ldots,r^T(\bullet)\right) \tag{3.6}$$

where U is a real valued utility function. Given the nature of **P1**, there are two obvious forms that this utility function might have. The first case is where utility is defined as the sum of the utility associated with the returns in each period i.e.,

$$U\left(r^1(\bullet),\ldots,r^T(\bullet)\right)=\sum_{t=1}^{T} u^t\left(r^t(\bullet)\right) \tag{3.7}$$

The second case is where utility is defined over the sum of the returns i.e.,

$$U\left(r^1(\bullet),\ldots,r^T(\bullet)\right)=U\left(\sum_{t=1}^{T} r^t(\bullet)\right) \tag{3.8}$$

Recall that, in decision theory, a utility function is used to reflect preferences for uncertain outcomes. In terms of **P1**, these two definitions of $U\left(r^1(\bullet),\ldots,r^T(\bullet)\right)$ can be incorporated into the objective as

U1
$$\max_{q^1,\ldots,q^T}\left\{\mathop{\mathrm{E}}_{a^1,\ldots,a^T}\left[\sum_{t=1}^{T} u^t\left(r^t(\bullet)\right)\right]\right\} \tag{3.9}$$

and

U2 $$\max_{q^1,\ldots,q^T}\left\{\underset{a^1,\ldots,a^T}{\mathrm{E}}\left[U\left(\sum_{t=1}^{T} r^t(\bullet)\right)\right]\right\} \tag{3.10}$$

though more general forms are certainly imaginable. For ease of exposition, it is assumed that the value of storage at the end of the horizon, $V\left(s^{T+1}\right)$, has been incorporated into the calculation of $r^T(\bullet)$. This does not lessen the generality of the discussion though, because $V\left(s^{T+1}\right)$ depends only on the state of the system, uncertainty, and decisions in T.

In decision theory, **U1** has been criticised because the axioms which support single stage decision problems do not extend so easily to multistage problems (Mossin, 1969; Spence and Zeckhauser, 1972; Kreps and Porteus, 1978; Kreps and Porteus, 1979). The criticism is due to the fact that lotteries with the same distribution of payoffs can have uncertainty resolving at different times, and this affects the DM's preferences, hence violating the independence axiom of von Neumann-Morgenstern utility theory (Mossin, 1969). Specifically, the DM's utility associated with uncertain consequences can rank alternatives differently depending on when uncertainty is resolved. Therefore, the decision in a given period, and hence the utility associated with it, may be dependent on the previous observed outcomes, and unknown potential outcomes (Kennedy et. al., 1994). Machina (1989) argues that expected utility can be modelled using temporal utility functions. Rae (1971) shows how to handle a case where utility is determined over the net present value of the sum of the returns, but does not account for system dynamics. Gilboa (1989) argues that additively separable utility functions aren't appropriate for representing DM preferences for sequential decisions problems at all. He proposes a formulation which considers the weighted average of utility and the variation in utility between consecutive periods. In terms of SDP, though, **U1** is additive and separable over time, and can therefore be maximised using SDP. The single period utility functions attempt to reflect a DM's preferences in each period, while the SDP decomposition handles the physical evolution of the system over time.

Objective **U2** is consistent with a decision theory approach to stochastic sequential decision making in that a utility function defined at a single point in time (at T) is being maximised. If the reservoir management problem were expressed as a decision tree, as in decision theory, then a value of utility would be associated with each terminal branch of the tree and the optimal path through the tree could be determined by folding back the tree. For small problems, solving a decision tree is tractable. For example, Kall and Wallace (1994) solve a decision tree using SDP for a 3-stage investment problem with a continuous wealth state variable and a single attribute utility function. The 'physical' state variable was the bank account in which funds were held, with only two bank accounts considered and all money was required held in a single account; splitting the money between accounts would have considerably increased the size of the decision tree. The non-linear utility function, defined over terminal wealth outcomes, is passed back to each stage in its functional form, given an assumption about the initial wealth. However, as the number of states and periods increases, the size of the decision tree can become impossibly large to analyse.

In the previous section, SDP was discussed as a technique for decomposing a problem (**P1**) into T sub-problems (**SDP1**). As stated earlier, for the principle of optimality to hold the objective of **P1** must be separable and monotone. Recall that the objective of **P1** being considered is

$$\max_{q^1,\ldots,q^T}\left\{\mathop{\mathrm{E}}_{a^1,\ldots,a^T}\left[U\left(\sum_{t=1}^{T} r^t(\bullet)\right)\right]\right\} \tag{3.1}$$

subject to the constraints for **P1**. If U is linear (i.e., there is no risk aversion) then returns in each t are just scaled by U. Therefore U can be moved inside the summation and:

$$\mathop{\mathrm{E}}_{a^1,\ldots,a^T}\left[U\left(\sum_{t=1}^{T} r^t(\bullet)\right)\right]=\left\{\mathop{\mathrm{E}}_{a^1,\ldots,a^T}\left[\sum_{t=1}^{T} U\left(r^t(\bullet)\right)\right]\right\} \tag{3.11}$$

With U linear (and positive), the policy which maximises the expected utility of the total returns is equivalent to that which maximises the expected utility of the single stage returns. So although U is defined over the returns in all stages, it can be incorporated directly into the calculation of the single stage returns. The resulting objective is just a linear transformation of that defined for **P1**, and therefore remains separable and can be solved using **SDP1**. Because $\mathop{\mathrm{E}}_{a^t}\left[\alpha\left(r^t\right)\right]\neq\alpha\mathop{\mathrm{E}}_{a^t}\left[r^t\right]$ for some constant $\alpha=U>0$, the policy that maximises the expected value of returns will also maximise the utility of the expected returns, which is the desired result. **SDP1** could therefore be solved in its original form and the value of U associated with the optimal policy determined afterwards. Note, too, that if uncertainty is removed and U is non-decreasing, but not necessarily linear, it is sufficient to maximise $\sum_{t=1}^{T} r^t(\bullet)$ (Nemhauser, 1966).

The scenario of interest here, though, is when U is non-linear, non-decreasing (in order to satisfy the monotonicity requirement of DP), and the outcomes are uncertain. As before, total utility is a function of all the returns over the planning horizon. With uncertain returns and U non-linear, it is not possible to move U inside the summation because

$$\mathop{\mathrm{E}}_{a^1,\dots,a^T}\left[U\left(\sum_{t=1}^{T} r^t(\bullet)\right)\right] \neq \mathop{\mathrm{E}}_{a^1,\dots,a^T}\left[\sum_{t=1}^{T} U\left(r^t(\bullet)\right)\right] \tag{3.12}$$

so U can not be used to scale the returns in each stage. The utility of all decisions must be considered simultaneously, so the objective is non-separable. Because U is non-linear, $\mathop{\mathrm{E}}_{a^t}\left[U\left(r^t\right)\right] \neq U \mathop{\mathrm{E}}_{a^t}\left[r^t\right]$, it is also not possible to move U outside the expectation and simply maximise the expected value of the returns; the value of utility associated with the policy that maximises the expected value of the total returns will not necessarily be that which maximises the expected utility of the returns.

In summary, **U1** can be implemented easily in SDP but has received criticism because the timing of uncertainty and decision making induces preferences which violate the independence axiom of expected utility theory. On the other hand, **U2** is consistent with decision theory but seemingly difficult to apply in a SDP context because the objective is not separable. In the next section, SUMDP is introduced as a modified version of **SDP1** which overcomes the non-separability of **U2**.

3.4 SUMDP model

Let U be a non-negative non-increasing utility function defined over the consequences of decisions made over the planning horizon. In terms of a decision tree, the 'consequence' at the end of the horizon is represented by the accumulated returns and the value of the state variable. At this stage, let the value of being at the final state be reflected by the terminal value function $f^{T+1}\left(s^{T+1}\right)$, which can be incorporated into $r^{T}(\bullet)$ without loss of generalisation. (This assumption about valuing s^{t+1} in units of $r^{t}(\bullet)$ will be relaxed later).

The problem to be solved is therefore

P2

$$\max_{q^1,\ldots,q^T}\left\{\mathop{\mathrm{E}}_{a^1,\ldots,a^T}\left[U\left(\sum_{t=1}^{T} r^t\left(s^t,q^t,a^t\right)\right)\right]\right\}$$

subject to

$$s^{t+1}=T_s^t\left(s^t,q^t,a^t\right)$$

$$q^t\in Q^t\left(s^t\right),\ s^t\in S^t \tag{3.13}$$

As discussed earlier, this objective is non-separable if U is non-linear, so a formulation like **SDP1** is not a valid approach to solving **P2**.

Kaye and Read (1998) show that **SDP1** can be reformulated so that the objective is separable. This is achieved by defining an *auxiliary* state variable, w^t, which is the accumulated returns (or wealth) up to the beginning of period t:

$$w^t=\sum_{k=1}^{t-1} r^k\left(s^k,q^k,a^k\right) \tag{3.14}$$

The level of wealth at the beginning of $t+1$ is therefore

$$w^{t+1}=w^t+r^t\left(s^t,q^t,a^t\right) \tag{3.15}$$

which is only dependent on the state of the system, the decision, and uncertainty, in t. The state transition equation for wealth has the general form

$$w^{t+1}=T_w^t\left(w^t,s^t,q^t,a^t\right) \tag{3.16}$$

and in this case,

$$w^{T+1}=\sum_{k=1}^{T} r^k\left(w^k,s^k,q^k,a^k\right) \tag{3.17}$$

given initial condition $w^1=0$. The feasible range of w^t, denoted by W^t, will depend on the bounds of $r^t\left(w^t,s^t,q^t,a^t\right)$, though w^t could be constrained to reflect, for example, requirements on the return in each period.

The (utility maximising) objective of **P2** can now be restated as

$$\max_{q^1,\dots,q^T}\left\{\underset{a^1,\dots,a^T}{\mathrm{E}}\left[U\left(\sum_{t=1}^{T} r^t\left(w^t,s^t,q^t,a^t\right)\right)\right]\right\}=\max_{q^1,\dots,q^T}\left\{\underset{a^1,\dots,a^T}{\mathrm{E}}\left[U\left(w^{T+1}\right)\right]\right\} \tag{3.18}$$

with the state transition equation for w^t and $w^1=0$. With the value of the storage at $T+1$ incorporated into $r^T(\bullet)$, **P2** can be restated as

P2a $$\max_{q^1,\dots,q^T}\left\{\underset{a^1,\dots,a^T}{\mathrm{E}}\left[U\left(w^{T+1}\right)\right]\right\}$$

subject to $$\left(w^{t+1},s^{t+1}\right)=T_{ws}^t\left(w^t,s^t,q^t,a^t\right)$$

$$q^t\in Q^t\left(s^t,w^t\right),\ s^t\in S^t,\ w^t\in W^t,\ w^1=0 \tag{3.19}$$

where $T_{ws}^t(\bullet)$ denotes the transition of $\left(w^t,s^t\right)$ to $\left(w^{t+1},s^{t+1}\right)$.

However, there are two consequences at the end of the planning horizon: accumulated returns *and* the storage level at T+1. Therefore, the general form of the utility function that needs to be considered is

$$U\left(w^{T+1},s^{T+1}\right) \tag{3.20}$$

An advantage of using $U\left(w^{T+1},s^{T+1}\right)$ is that w^{T+1} and s^{T+1} do not need to be expressed in the same units; $U\left(w^{T+1},s^{T+1}\right)$ just needs to reflect the DM's preferences with respect to combinations of w^{T+1} and s^{T+1}. For some problems, though, comparing outcomes using like units is reasonable e.g., **P1** and **SDP1** used $U\left(w^{T+1},s^{T+1}\right)=w^{T+1}+f^{T+1}\left(s^{T+1}\right)$. (An interesting result shown in is that, for a specific form of non-separable utility function, the optimal policy in each period is independent of the auxiliary variable).

The terminal value function is therefore defined over the states of the system at the end of the planning horizon, so

$$f^{T+1}\left(w^{T+1},s^{T+1}\right)=U\left(\sum_{t=1}^{T} r^t,s^{T+1}\right)=U\left(w^{T+1},s^{T+1}\right) \tag{3.21}$$

Problem **P2** can now be formulated as a SDP with state variables for the system state and the accumulated return. The problem to be solved for t=T-1,…,t is

SDP2
$$f^t\left(w^t, s^t\right) = \max_{q^t} \left\{ \mathop{\mathrm{E}}_{a^t}\left[f^{t+1}\left(w^{t+1}, s^{t+1}\right)\right]\right\}$$

subject to:
$$\left(w^{t+1}, s^{t+1}\right) = T_{ws}^t\left(w^t, s^t, q^t, a^t\right)$$

$$q^t \in Q^t\left(s^t, w^t\right),\ s^t \in S^t,\ w^t \in W^t$$

$$w^1 = 0 \tag{3.22}$$

Using backwards recursion, the expected 'utility to go' in any period is induced from the expected utility to go from the subsequent period; utility is not derived from combining utility functions over multiple time periods.

The fundamental difference between **SDP1** and **SDP2** is that in the latter the state space in each stage has been increased from S^t to $S^t \times W^t$, and $f^t\left(w^t, s^t\right)$ is measured in units of utility rather than monetary units, which affects the way that $f^t\left(w^t, s^t\right)$ is calculated. This latter difference potentially increases the applications of SUMDP because U is not required to be defined over commensurable variables. Adding the auxiliary variable to the state space has removed the process, and requirement, of comparing decisions in t according to the value of $\mathrm{E}\left[r^t\left(w^t, s^t, q^t, a^t\right) + f^{t+1}(\bullet)\right]$. Decisions are now compared by evaluating the expected value of $f^{t+1}(\bullet)$ for the values of the state variables in $t+1$, with the return embedded in the definition of the state variable. SUMDP, of itself, does not place any constraint on the type of utility function other than those required to satisfy the minimal requirements of dynamic programming as defined by Nemhauser (1966).

For **U2**, the attitudes of the DM towards w^{t+1} and s^{t+1} evolve over the time periods as the recursive relation is evaluated, so explicit attitudes towards them are not considered. Whether this is applicable depends on the nature of the problem, particularly the length of the time periods and the planning horizon. In a managerial context, for example, performance might be assessed during the planning horizon, as well as at the end. If the periods over which performance is assessed is from x to y, with $y>x$, then total utility could be expressed as

$$U\left(r^1(\bullet), \ldots, r^T(\bullet)\right) = U\left(\sum_{t=x}^{y} r^t(\bullet), \sum_{t=1}^{T} r^t(\bullet)\right) \tag{3.23}$$

If the DM is concerned with n consequences (attributes) which are the sum of additively separable 'returns' in each period, and U can be defined over these, then n auxiliary state variables can be added to the problem. This observation is not specific to reservoir management applications of SDP. In practice, this process will result in a problem with a state space increasing exponentially in n, as identified earlier. However, bounds on w^t, the form of $r^t(w^t, s^t, q^t, a^t)$, and the form of U may restrict the state space and decisions required to be considered at each stage. This study deals with the case where a single auxiliary variable is added. In chapters 5 and 6, techniques for reducing the state space required to be searched are presented for the specific problems therein (and where w^t is defined as the accumulated returns).

3.5 Related techniques

The concept of an 'accumulated value' state variable is not new to the dynamic programming literature, nor is the concept of utility maximisation. However, few techniques combine utility maximisation and stochastic sequential decision problems. This section discusses some relevant techniques and modelling approaches.

The procedure of augmenting the state space to overcome non-separability has been discussed in general texts (e.g., Nemhauser, 1966; Kall and Wallace, 1994) and in publications on specific application areas (e.g., Yeh, 1985; Sniedovich, 1989). Nemhauser (1966) describes a (deterministic) terminal optimisation problem as one where the objective is to maximise a function of the final output state, which is a special case of a problem maximising the sum of the returns.

$$f^1(s^1) = \max_{q^1,\ldots,q^T} \{U(s^{T+1})\}$$

subject to $$s^{t+1} = T_s^t(s^t, q^t) \qquad \forall t \quad (3.24)$$

To represent the problem as a maximisation of the sum of the returns, let $r^t(s^t, q^t) = 0 \;\; \forall t < T$ and $r^T(s^T, q^T) = U(T_s^t(s^t, q^t)) = U(s^{T+1})$ for $t = T$, which gives the desired relationship

$$\sum_{t=1}^{T} r^t(s^t, q^t) = U(s^{T+1}) \qquad t < T \quad (3.25)$$

The original problem can therefore be restated as

$$f^1(s^1) = \max_{q^1,\dots,q^T} \left\{ \sum_{t=1}^{T} r^t(s^t, q^t) \right\}$$

subject to $$s^{t+1} = T_s^t(s^t, q^t) \qquad \forall t \qquad (3.26)$$

which is decomposable into T sub-problems of the form

$$f^t(s^t) = \max_{q^t} Q^t(s^t, q^t)$$

subject to $$s^{t+1} = T_s^t(s^t, q^t) \qquad (3.27)$$

where $Q^t(s^t, q^t) = r^t(s^t, q^t)$ for $t=T$ and $Q^t(s^t, q^t) = r^t(s^t, q^t) + f^{t+1}(T_s^t(s^t, q^t))$ $\forall t < T$. With uncertainty (independent in each period), the same approach results in sub-problems of the form

$$f^t(s^t) = \max_{q^t} \mathop{E}_{a^t}\left[Q^t(s^t, q^t, a^t)\right]$$

subject to $$s^{t+1} = T_s^t(s^t, q^t, a^t) \qquad (3.28)$$

where $Q^t(s^t, q^t, a^t) = r^t(s^t, q^t, a^t)$ for $t=T$ and

$Q^t(s^t, q^t, a^t) = r^t(s^t, q^t, a^t) + f^{t+1}(T_s^t(s^t, q^t, a^t))$ $\forall t < T$.

Clearly, **SDP1** has this form. SUMDP, as stated in **SDP2**, is just a terminal optimisation problem where the terminal states are w^{T+1} and s^{T+1} which evolve according to the transition functions $T_w^t(\bullet)$ and $T_s^t(\bullet)$.

Ranatunga (1995) applied a version of SUMDP to the purchase and sale of forward contracts so a risk averse DM owning thermal plant with inter-temporal uncertainty could hedge against price uncertainty. The state variables were the number of units committed in t, the accumulated return, and the (Markov) state of prices. The terminal value function was defined as

$$f^{T+1}(\bullet) = U(r^1, s^1, \dots, r^T, s^T) = u\left(\sum_{t=1}^{T} r^t\right) = u(w^{T+1}) \qquad (3.29)$$

This is a single state terminal optimisation problem with the state being the accumulated wealth. Preferences toward the state of the system at the end of the planning horizon were not considered. This is a reasonable assumption for a short-term problem because the commitment level of the station can be constrained to be equal at the beginning and end of the day, and can only be in one of a few discrete states. For problems such as reservoir management, though, this assumption is less satisfactory because it is storage that is being managed, and hence the trade-off between storage and wealth needs to be explicit.

Commenting on the yield management model of Krautkraemer et. al. (1992), Kennedy et. al. (1994) argue that an additive utility function of the form in **U1** does not explicitly account for variation between periods. They suggest that a DP approach using an objective such as that of Gilboa (1989) could be used to reflect this. Another state variable is proposed to record the expected utility from returns in the previous period. The decision in t therefore depends on the moisture level and the expected utility 'earned' in t-1. In the context of a ship routing problem, Psaraftis & Tsitsiklis (1993) noted that in order to extend their DP model to handle a time window with penalties for early and/or late arrival, the state space required the inclusion of the time dimension, though they did not elaborate.

In a reservoir management setting, Sniedovich (1980) considered a constraint on the variance of the total number of failures. His study was motivated by the following conclusion in Rossman (1977): "*... the variance of the number of failures over the reservoir's life cannot be considered as either an objective function or a constraint because the resulting mathematical expression is not separable and thus dynamic programming could not be used ...*" (p.254). The same argument could be (incorrectly) made with respect to a utility function defined over the returns accumulated over the reservoir's life. Hence, the approach taken by Sniedovich is relevant. If the failure in a particular period is denoted by the function $x^t\left(s^t, q^t, a^t\right)$ then the total number of failures over the life of the reservoir is

$$x\left(s^1, q^1, a^1, \ldots, s^T, q^T, a^T\right) = \sum_{t=1}^{T} x^t\left(s^t, q^t, a^t\right) \tag{3.30}$$

where $x^t(\bullet)=1$ if a 'failure' occurs in period t and 0 otherwise. The problem involves maximising the expected value of total benefits while ensuring that the variance of

$x^t(\bullet)$ does not exceed a given value *u*. While the expected benefit is additively separable, the variance of the failures is not; it is a function of all the failures which occur over the planning horizon, as correctly observed by Rossman (1977).

To model a constraint of this sort, Sniedovich introduces a variable y^t which is the total number of failures up to period *t*:

$$y^t = \sum_{m=1}^{t-1} x^m\left(s^m, q^m, a^m\right) \qquad 1 < t \le T+1 \quad (3.31)$$

with $y^t = 0$ (note the correspondence to the definition of w^t in SUMDP). Because $x^t(\bullet)$ is 0 or 1 in any given *t*, a given y^t will take a value in the range {0, *t*-1} and the value of y^{t+1} can be determined by

$$\begin{aligned} y^{t+1} &= T\left(s^t, q^t, a^t, y^t\right) \\ &= y^t + x^t\left(s^t, q^t, a^t\right) \end{aligned} \qquad 1 < t \le T \quad (3.32)$$

with the total number of failures at the end of the horizon being $y^{T+1} = \sum_{t=1}^{T} r^t\left(s^t, q^t, a^t\right)$. Utilising the definition of variance, a Lagrangian problem is defined with the reliability constraint incorporated into the objective. A two-state SDP is formulated with a terminal value function $F^{T+1}(s^t, y^t) = -\lambda\left(y^{T+1} - u\right)^2$ where *u* is the mean number of failures and λ is the penalty term on the failure constraint which was moved into the objective. The Lagrangian problem is redefined again as a function only of λ' with *u* treated as a decision variable. Solutions are the expected benefit E[*B*] (given an initial storage) and the value of the variance (*v*) on the RHS of the constraint associated with the optimal solution derived using λ. Each solution for a fixed λ is found by repeatedly solving the two-state SDP over the range of *u*. As λ increases, E[*B*] and *v* decrease, or in other words, a higher value of *v* increases E[*B*], which is what would be expected. Rather than deal with preferences explicitly, as in SUMDP, this approach produces a mean/variance trade-off curve.

As stated earlier, the additional state variable in SUMDP is used to make objective **U2** separable, though there are alternatives. Carraway et. al. (1990) describe a DP approach to multicriteria deterministic sequential decision problems. Each decision (arc) has two outcomes, and the objective is to maximise a utility function *U(x,y)*, which is defined as a function of these outcomes at the end of the horizon (the last node of the network). They show that if *u(x,y)* is used to evaluate decisions (arcs) from each node, a sub-optimal path will be produced. This is the same argument motivating SUMDP. Whereas SUMDP utilises Bellman's principle of optimality, Carraway et. al. (1990) utilise a weak principle of optimality: *"an optimal path must be composed of subpaths that can be part of an optimal path"* (p. 98). This is an alternative to using *"an optimal path must be composed of optimal subpaths"* which underlies DP in general and SUMDP in particular.

With the weak principle of optimality, subpaths are not necessarily optimal with respect to the actual objective function *u(x,y)*, where *x* and *y* are the criteria being considered in selecting a path. Instead, a "refining local preference relation" is used to determine preferences at each node. Preferences are induced from *u(x,y)* and utilise the bounds on the criteria at each node. Henig (1990) also utilises bounds on returns, showing that extreme partial solutions are optimal and can be determined analytically. However, his approach requires a rather restrictive set of assumptions (e.g., uncertainty resolved after all decisions made) which exclude problems such as reservoir management and stochastic route choice problems.

Despite the theoretical weakness of using **U1**, $\max_{q^1,\ldots,q^T}\left\{\mathop{\mathrm{E}}_{a^1,\ldots,a^T}\left[\sum_{t=1}^{T}u^t\left(r^t(\bullet)\right)\right]\right\}$, Krautkraemer et. al. (1992) describe an SDP application with an objective of **U1** in the context of an agricultural planning problem. Each period corresponds to a year, and utility functions are defined over the returns in each year. The state variable is the moisture content of the land, the decision variable is whether to crop or fallow, the return is monetary, and there is uncertainty in the weather conditions. They argue that the biases resulting from explicitly assuming risk neutrality are considered to be more than those arising from violating the independence axiom. They concluded that the influence of risk on optimal DP solutions required more research because, at least for agricultural problems, DM's can be risk averse towards outcomes within and between years.

Levitt and Ben-Israel (2001), developing on the work of Ben-Tal and Ben-Israel (1991), consider an additive utility function like **U1**. Whereas Krautkraemer et. al.

(1992) use additive utility functions, Levitt and Ben-Israel consider the certainty equivalent of the returns. The utility function from which the certainty equivalent is derived is therefore measured in the same units as the returns e.g., a quadratic function with a risk parameter as is often used in financial planning. At each stage, the return is added to the certainty equivalent of the 'cost to go'.

With an objective such as **U2**, $\max_{q^1,\dots,q^T}\left\{\mathop{\mathrm{E}}_{a^1,\dots,a^T}\left[U\left(\sum_{t=1}^{T} r^t(\bullet)\right)\right]\right\}$, utility is defined as a non-separable function of the returns over the entire planning horizon. SUMDP handles this by redefining the problem as a terminal optimisation problem. The returns in each period, apart from the last period, are set to zero, and the expected utility function is induced from period to period via the state transition functions. Therefore, the returns in t do not impact on the 'cost to go' in $t+1$ as in a conventional DP formulation where the non-zero returns are added to the cost-to-go from the next stage.

3.6 Summary

SUMDP combines the utility and stochastic dynamic programming to essentially provide a method for evaluating a decision tree that is more efficient than complete enumeration due to the stage-wise decomposition. Aspects from both are required to approach finite horizon stochastic sequential decision problems where the dynamics of a system must be considered and where the DM has non-linear preferences for the consequences of decisions. One of the requirements of DP is that the objective must be separable. The typical objective of maximising expected value is separable, but, according to utility theory, also implies that the DM is risk neutral, and this may not always be the case. Indeed, the reservoir management literature is littered with descriptions of 'risk averse' operators and firms.

Utility functions can be used to reflect a DM's preferences towards multiple uncertain consequences at a single point in time, and the form of these utility functions reflects the DM's attitude to risk. Extending utility functions to stochastic sequential decision problems is usually achieved by aggregating single period utility functions. This can result in a non-optimal policy as well as being problematic from a practical perspective because utility functions must be defined for every period. It has been argued that this is in fact a desirable property because the DM has the flexibility to define different preferences for different periods.

SUMDP extends SDP to handle the non-separable objective implied by defining a utility function over the consequences at the end of the planning horizon. This is achieved by defining an auxiliary variable as the 'accumulated returns', and assuming that the DM's preferences are defined over the sum of the returns. As a result, preferences are defined over the 'accumulated returns' at the end of the planning horizon; utility is defined at a single point in time (at the end of the planning horizon). SUMDP therefore does not require that all periods, and decisions within those periods, be considered simultaneously. In fact, SUMDP can be extended to a range of problems where the accumulated return is a function of a subset of decisions in preceding stages, as long as a stage is not returned to. This occurs naturally in stochastic sequential problems with a temporal dimension, but also means that SUMDP can be applied to problems which can be represented as a (stochastic) directed acyclic network.

The practical and theoretical applications of such a technique are broad due to the relative simplicity of the conditions required to extend a SDP formulation to handle the 'utility maximising' formulation. A drawback, though, is that the auxiliary variable increases the number of states evaluated at each stage. Methods for reducing the impact of this are dependent on the characteristics of the problem.

The problem considered in this thesis is that of medium-term reservoir management. It is a well researched problem, with SDP being a suitable modelling technique due to its relative flexibility and ability to model the dynamics of reservoir operation and inflow uncertainty. However, SDP methods are usually defined with an objective of maximising expected value; if risk is considered, it is normally incorporated into the constraints of the problem (Yeh, 1985). SUMDP develops on existing reservoir management models which incorporate risk because the objective is to maximise the utility of accumulated returns and storage at the end of the planning horizon. Furthermore, electricity market deregulation has affected the environment in which many reservoir systems are managed. The return from a release is now a revenue rather than a cost and can be derived in a variety of ways depending on the assumptions made about the nature of competition. There are very few reservoir management models which address the issues involved in 'deregulated' reservoir management, and none that incorporate risk attitudes using a utility function.

The layout of the remainder of this thesis is as follows.

- Chapters 4 and 5 present theoretical and implementation issues for SUMDP applied to a reservoir management in a regulated (or centralised) system. presents experimental results.
- Chapters 7 and 8 present theoretical/implementation issues and experimental results for reservoir management in a deregulated (or decentralised) environment.
- discusses some extensions to SUMDP when modelling reservoir management in a deregulated environment.
- departs from the reservoir management theme and discusses SUMDP in the context of stochastic route choice problems, which are a form of sequential stochastic decision problem for which utility maximisation is relevant but has received relatively little attention.
- presents conclusions.

Chapter 4

SUMDP and Reservoir Management in a Regulated Electricity Market

4.1 Introduction

In the previous chapters, SUMDP was introduced as an approach to stochastic sequential decision problems where the objective is to maximise utility. This technique can be applied to reservoir management, which involves planning future releases given uncertainty about future inflows into the reservoir. For classical reservoir management in electricity generation systems, the objective of stochastic models is typically to minimise the expected cost of satisfying demand from non-hydro generation. This is consistent with the objective used the 'regulated' or Government-owned electricity system as existed in New Zealand prior to deregulation. However, this objective implicitly assumes that the reservoir is operated in a risk neutral manner, and this assumption may not always be appropriate. The nature and impacts of the stochasticity inherent in this problem, added to the fact that it is relatively well defined, make it a suitable candidate for analysing the impact of utility functions on system performance.

This chapter discusses how SUMDP can be applied to a classical medium-term reservoir management problem, as faced when managing hydro reservoirs and thermal plant in the New Zealand electricity system prior to deregulation. The layout of this chapter is as follows:

- The SUMDP formulation for the problem is presented in Section 4.2.
- A two-state transition function is discussed in Section 4.3.
- Utility functions are discussed in Section 4.4.
- Conclusions are presented in Section 4.5.

Throughout this chapter, concepts are illustrated using a representation of the New Zealand system data used to derive the experimental results in , as well as using illustrative data.

4.2 SUMDP and 'regulated' reservoir management

The situation considered here is that faced by a DM managing a reservoir and a number of other electricity supply sources such as run-of-river hydro stations and thermal sources. Demand must be met in each period and there are bounds on release and on storage. The inflows experienced in each week are uncertain. The objective is to dispatch the system to maximise the expected value of storage and benefits (negative costs) at the end of the horizon. This situation is representative of that faced by a Government or similar entity that manages the supply of electricity to consumers. The same circumstances could be faced by a firm (or agent), operating in a regulated or deregulated market but being required to satisfy a demand target. Instead of meeting national demand, the agent would meet a demand target specified by the principal.

In reservoir planning problems, it is necessary to value terminal storage (using a water value function) to ensure that adequate storage is carried through to the next planning horizon. Reservoir management models (e.g., Yang, 1995; Scott, 1997) use a water value function, $V\left(s^{T+1}\right)$, which is defined over the ending storage levels and increases at a non-increasing rate as s^{T+1} increases. In a classical primal DP model, $V\left(s^{T+1}\right)$ is used as the terminal value function (in dual DP (Read, 1989) the marginal water value function, or $\partial V\left(s^{T+1}\right)/\partial s^{T+1}$, is used). In terms of a conventional DP formulation, the 'cost to go' in the final period is defined as $f^{T+1}\left(s^{T+1}\right)=V\left(s^{T+1}\right)$. $V\left(s^{T+1}\right)$ can be provided from a higher level model which has a longer time frame (say 20 years), or from solving a model to equilibrium. (A conventional SDP approach (**SDP1**) to the reservoir management problem **P1** was introduced in).

In SUMDP, the terminal value function reflects the desirability of achieving different levels of storage *and accumulated wealth* at the end-of-horizon, so

$$f^{T+1}\left(w^{T+1},s^{T+1}\right)=U\left(w^{T+1},s^{T+1}\right) \tag{4.1}$$

An alternative is to substitute $V\left(s^{T+1}\right)$ for s^{T+1} in the definition of $U\left(w^{T+1},s^{T+1}\right)$, which implies both terms of the utility function are in monetary units. This reduces generality but may aid DMs in defining a utility function. Neither alternative makes any difference to the computational effort of the technique.

SUMDP was introduced in . The SDP model of interest here, referred to as **RM-R** (**R**eservoir **M**anagement – **R**egulated) is essentially the same as that described in , and can be described as follows

RM-R

$$f^{t}\left(w^{t},s^{t}\right)=\max_{q^{t}}\ \mathop{\mathrm{E}}_{a^{t}}\left[f^{t+1}\left(w^{t+1},s^{t+1}\right)\right] \qquad \forall t$$

subject to:

$$w^{t+1}=w^{t}+B^{t}\left(q^{t}\right) \qquad \forall t$$

$$s^{t+1}\leq s^{t}-q^{t}+a^{t} \qquad \forall t$$

$$\underline{q}^{t}\leq q^{t}\leq\overline{q}^{t} \qquad \forall t$$

$$\underline{s}^{t}\leq s^{t}\leq\overline{s}^{t} \qquad \forall t$$

$$w^{1}=0 \tag{4.2}$$

where

T	is the finite set of periods (t) in the planning horizon.
w^{t}	the accumulated cost at the beginning of t.

s^t — the reservoir storage level at the beginning of t (measured in units of electricity e.g. MWh or GWh).

$f^t(w^t, s^t)$ — is the expected end-of-horizon utility for a given wealth and storage combination in period t.

a_t — the stochastic inflow into the reservoir during t. It is assumed that a distribution of inflows can be described (in this case based on historical data), and that these inflows are not correlated over time. See Yang (1995) for treatment of the correlated case.

q_t — the release during t which is the decision variable in each stage and state. The optimal release in a given period, $\hat{q}^t(w^t, s^t)$, is a function of w^t and s^t i.e., $\hat{q}^t(w^t, s^t) = \underset{\underline{q}^t \le q^t \le \overline{q}^t}{\arg\max} \ \mathrm{E}\left[f^{t+1}(w^{t+1}, s^{t+1})\right]$.

$B^t(q^t)$ — is a function describing the cost from release, and is a specific form of the more general $r^t(w^t, s^t, q^t, a^t)$. The benefit function is different in each period because it incorporates parameters such as demand and the cost of meeting demand; it does not depend on the level of the state or inflow variables.

$\underline{s}^t, \overline{s}^t$ — are the lower and upper bounds on storage in t.

$\underline{q}^t, \overline{q}^t$ — are the lower and upper bounds on release in t.

The SDP is solved here using discrete dynamic programming, though it could be solved by other SDP variants such as Dual DP (Read & George, 1995) or Constructive DP (Travers and Kaye, 1997), as exemplified by the work of Craddock et. al. (1999). As discussed in the previous chapter, the aspects of SUMDP which differ from a conventional DP approach are the incorporation of the benefit function in to the transition of w^t to w^{t+1}, and the utility function $U(w^{T+1}, s^{T+1})$. These two aspects of the problem are discussed in sections 4.3 and 4.4, respectively. (Issues relating to the solution technique and other implementation issues are discussed in further detail in).

4.3 The benefit function

The type of regulated electricity system considered here is one where a single entity, or DM, manages a number of generating plant so as to meet demand in each period. The case considered here is where the DM manages a single reservoir and a number of 'non-reservoir' plant e.g., coal-fired, oil-fired and geothermal plant. The return from release in each period is the cost of satisfying an estimate of demand. In order to estimate this cost, demand and supply must be modelled, as discussed below.

4.3.1 Demand and supply

Weekly demand can be represented by a load duration curve (LDC), which reflects a cumulative probability distribution for load, or equivalently, the proportion of the week for which each possible load level will last. For example, extremely high demand will only occur for a few hours each week, and there will be some minimum load that must be satisfied for every hour of the week. The LDC is essentially a continuous curve, but for the purposes of the optimisation, must be approximated in some way. A common way is to divide the LDC into discrete time segments, termed sub-periods, during which the demand is approximated using a single demand curve.

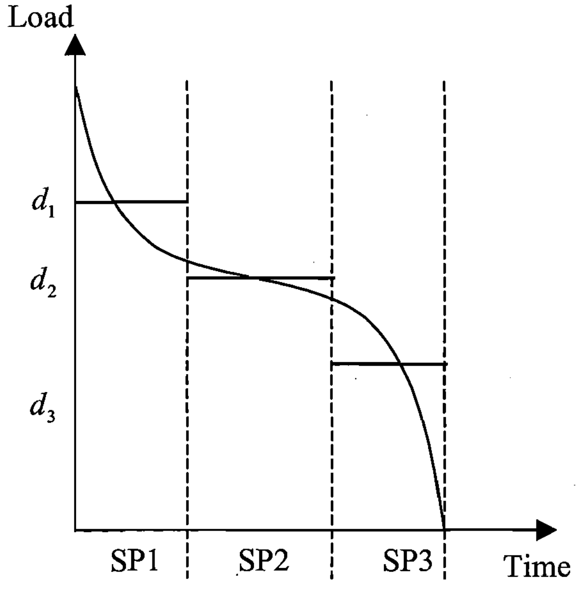


Figure 4.1: Approximating LDC with multiple 'fixed' demand curves

Figure 4.1 illustrates a weekly LDC and a 3-subperiod approximation where the demand in each sub-period is approximated by a fixed (or constant) quantity. When demand is approximated by a fixed demand level, it is assumed to be perfectly inelastic, so the quantity demanded does not change as the price changes. More generally, there will be some price, albeit extremely high, at which any party will be willing and able to decrease its quantity demanded.

Demand is usually modelled by a function describing the relationship between price and quantity. Examples include constant, linear, quadratic or constant elasticity demand curves. Demand for a constant quantity of electricity can be represented mathematically (and in inverse form) as

$$g(p) = d \tag{4.3}$$

where d is a constant. A linear demand curve can be represented by the function

$$p(g) = p_0 + p_1 g \tag{4.4}$$

where p_0>0 and p_1<0 (t superscripts have been excluded). Here, $p(g)$ reflects the price, per unit, that consumers are prepared to pay for some quantity g. For the constant elasticity case, if the price elasticity of demand is defined as

$$\varepsilon = -\frac{\partial g}{\partial p}\frac{p}{g} \tag{4.5}$$

then the inverse market demand curve can be described as

$$p(g) = p_0\left(\frac{g}{g_0}\right)^{\frac{1}{\varepsilon}} \tag{4.6}$$

where p_0 and g_0 are reference points for price and generation through which the demand curve passes. The linear demand curve can also be defined in this way i.e., as a line with a negative slope that passes through a reference point.

The remainder of this analysis for this regulated case assumes fixed demand (as does the deregulated case discussed in later chapters). Using linear and constant elasticity demand curves would potentially aid in making the model more realistic. These can be implemented with relative ease and do not jeopardise the SDP algorithm because the form of the resulting benefit function remains convex in the release level (see for the linear demand case).

Demand is met in each period by a combination of generation from reservoir release and thermal plant. The non-hydro plant are referred to as 'thermal plant', and have the (assumed) characteristic that they will generate any quantity specified by the DM and that inter-temporal linkages do not need to be explicitly modelled. The bounds on generation for each plant in each period are assumed known, so

$$\underline{g}_i^t \leq g_i^t \leq \overline{g}_i^t \qquad \forall i \qquad (4.7)$$

where g_i^t is the generation of thermal station i in period t, and $\underline{g}_i^t, \overline{g}_i^t$ are the lower/upper generation bounds. Total system generation in a period is therefore comprised of the electricity provided from reservoir release and the contributions from I other power stations:

$$g_S^t = q^t + \sum_{i=1}^{I} g_i^t \qquad (4.8)$$

Because the interaction between supply and demand was not a primary focus (as in Scott (1997) for example), demand in each period was assumed to be constant. In fact, for a given linear demand curve, a fixed demand curve can be defined such that the prices resulting from release closely approximates those obtained using a linear demand. Furthermore, Scott (1997) argued that the choice between a linear and constant elasticity demand curve was essentially arbitrary, so long as the equilibrium wasn't too far away from the reference points. With fixed demand, then, the load met by the other stations, given release q^t, is therefore $d^t - q^t$, and the cost of generating this amount can be calculated by considering the generation costs of the thermal plant.

Release in a given period must be feasible with respect to storage bounds. Storage bounds can be handled in a variety of ways, depending on when inflows are assumed to arrive in the reservoir. It is assumed, rather conservatively, that release q^t is actually made, not only before a^t is known, but before any inflow occurs at all. Thus a feasible release also satisfies

$$q^t \leq s^t - \underline{s}^t \qquad (4.9)$$

(Equivalently, the minimum release bound can be represented by $\overline{q}^t$ being a non-decreasing function of the storage level and $q^t \leq \overline{q}^t(s^t)$). Using this approach avoids having to handle the situation where the release exceeds the inflow, resulting in a negative storage because $s^{t+1} < \underline{s}^{t+1}$. At the other extreme, s^{t+1} is bounded at $\overline{s}^{t+1}$, so the quantity $\left(s^t - q^t + a^t\right) - \overline{s}^t$ is treated as being spilled and at no cost. The benefit to consumers would remain constant since the extra release is essentially spilled, and it is likely that excessive spill will be undesirable. Though only likely to occur when storage is high, release decisions which imply excessive spill could be penalised in order to reflect non-electricity related costs such as the environmental costs of flooding. If these costs were convex, which seems a realistic assumption, the cost curve would start to rise again as spill increases.

In terms of thermal generation, each thermal plant has a known marginal fuel cost, c_i, and the stations are dispatched in order of marginal cost, from lowest to highest. These capacities and marginal costs can be used to construct a stepped supply curve which represents, for each feasible generation level, the lowest (unit) cost of supplying another unit of generation. To illustrate some of the concepts in this chapter a hypothetical system with a single reservoir and 3 stations will be referred to. The capacities and marginal costs of the stations and the reservoir release bounds are shown in Table 4.1 and the stepped supply curve is illustrated in Figure 4.2.

i	$\underline{g}_i^t$	$\overline{g}_i^t$	c_i
Station 1	0	6	\$20
Station 2	0	6	\$40
Station 3	0	3	\$50
	$\underline{q}^t$	$\overline{q}^t$	
Reservoir	0	10	

Table 4.1: Illustrative system data

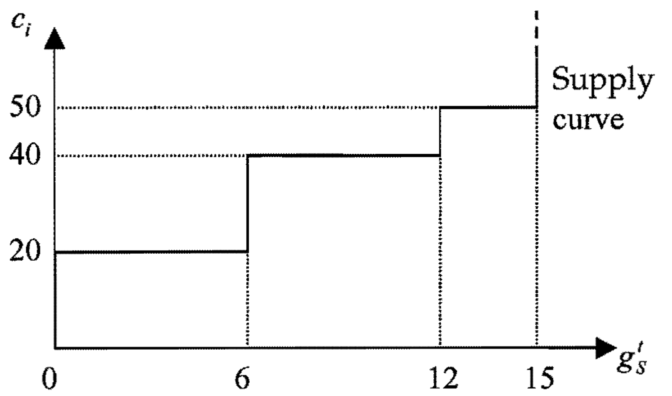


Figure 4.2: Supply curve

The MWh costs are of a similar scale to those of stations in the New Zealand electricity system. When a single demand curve is used, outputs from this analysis are multiplied by 168 to represent the same dispatch occurring for the entire week (168 hours). Note that in order to ensure an intersection between the supply and demand curves exists, additional steps can be added to the supply curve. These would have substantially higher marginal costs than the existing thermal plant and reflect the cost of non-supply, or shortage. In the experimental results discussed in Chapter 6, for example, the supply curve is augmented with a single 'shortage' step (or dummy thermal station) with a marginal cost of \$500/MWh. This shortage cost has also been used in other reservoir management models for the New Zealand system (e.g. Yang, 1995), and is approximately twenty times larger than the average marginal cost of generation.

4.3.2 Deriving the benefit from release

Recall that demand for electricity in the period is approximated with a constant demand level. This demand is met by a combination of reservoir release, q^t, and thermal generation and the cost of meeting the demand not met by reservoir release can be expressed as

$$C^t\left(q^t, d^t, \mathbf{g}^t, \mathbf{c}^t\right) = \sum_{i=1}^{I} g_i^t c_i^t \tag{4.10}$$

where $\mathbf{g}^t$ is the vector of thermal generation levels and $\mathbf{c}^t$ is the vector of thermal marginal costs. The largest cost is incurred when $q^t = 0$, and, ignoring storage bounds, the lowest cost is incurred when $q^t = \overline{q}_t$. Note that release is discretised at K points, with a particular discrete release being indicated with the subscript k. This subscript is only included when necessary.

Figure 4.3 illustrates the stepped supply curve, as well as how the demand (fixed) of 12MW is met when $q_k^t = 3$. Station 1 is fully dispatched and station 2 is partially dispatched, so $g_1^t = \overline{g}_1^t$ and $\underline{g}_2^t \leq g_2^t \leq \overline{g}_2^t$. Station 3 is not dispatched because $c_3 > c_2$ and $g_2^t < \overline{g}_2^t$. If the electricity required from station 2 changes, as a result of increasing release, say, and g_2^t remains within its bounds, station 2 will remain the cheapest supplier of electricity. Station 2 is referred to as the marginal station because it is supplying the 'last' unit of generation, and at the marginal production cost c_2.

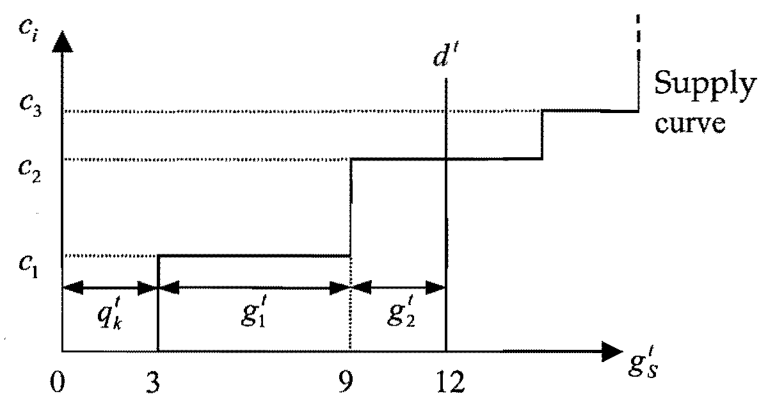


Figure 4.3: Supply/demand equilibrium (fixed demand)

An increase in q^t of 1MW shifts the supply curve to the right, reducing the generation of the marginal station and reducing the total cost by the marginal cost of that station. Thus, an additional 1MW of hydro generation will reduce g_i^t by 1MW for the marginal station.

An alternative, and equivalent, way to consider this process is to create a residual demand curve (or demand curve for release), which is the demand curve faced by the reservoir given the potential output from the thermal stations. It is derived by subtracting the thermal supply curve from the demand curve. When demand is fixed at quantity d^t, the RDC is the mirror image (vertical axis) of the supply curve truncated at d^t, which is shifted so that d^t becomes the y-axis through the origin. Figure 4.4 shows the RDC using the supply and demand curves in Figure 4.3.

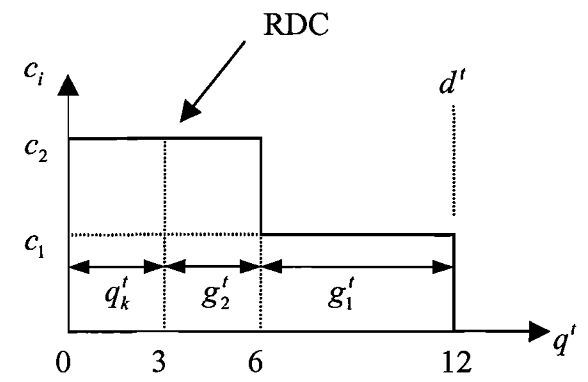


Figure 4.4: Residual demand curve (fixed demand)

The cost of meeting demand is the area under the RDC curve between q^t and d^t, or

$$C^t(q^t, d^t, \mathbf{g}^t, \mathbf{c}^t) = \int_{q^t}^{d^t} RDC^t(q^t)\partial q^t \tag{4.11}$$

Figure 4.5 illustrates $C^t(\bullet)$ for the range of feasible q^t using illustrative $\mathbf{g}^t$ and $\mathbf{c}^t$ values and with $d^t = 12\text{MW}$. As q^t increases, the amount of thermal generation decreases, and so does the cost. A change in the slope of the curve corresponds to moving to an adjacent step in the RDC with an equal or lower marginal cost, so $C^t(\bullet)$ is convex.

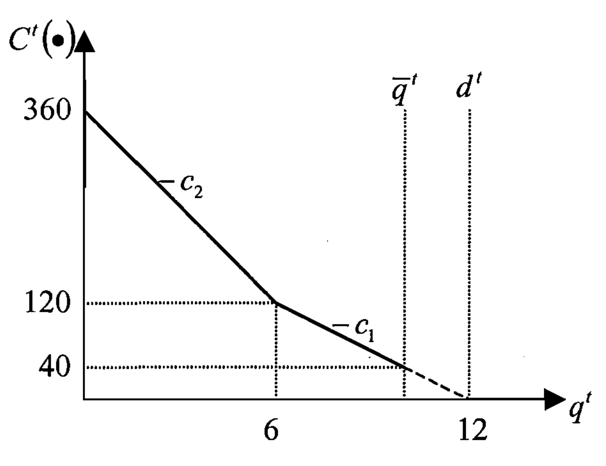


Figure 4.5: Cost of meeting demand (fixed demand)

Ignoring storage bounds, if $\bar{q}^t \geq d^t$ then it will be feasible to satisfy demand from hydro generation alone, and the total cost will be $C^t(\bullet) = 0$. In this model, $\bar{q}^t \leq d^t$, though the case where $q^t > d^t$ is not unrealistic.

This approach can be generalised. For example, a similar convex cost function is derived by Pereira, Campodonico, and Kelman (1998), who describe a probabilistic model which calculates the expected cost of satisfying demand for the range of reservoir release levels. The model takes account of forced thermal plant outages and does not require any approximations of the LDC. Calculating the cost when hydro replaces the merit order position of the different thermal stations produces the cost function. The cost curve they describe is convex and piecewise linear, which is the same general form as the cost curves described in the following sections. A similar approach was used in PRISM, which was a Dual DP based model (Read and George, 1990) used for planning of electricity supply in New Zealand.

In the context of satisfying a national demand where the consumers, via the Government, say, own the generation stations, $B^t(q^t)$ should represent the (national) benefit to electricity users from consuming d^t, $NB(d^t)$, less the cost of producing d^t:

$$B^t(q^t) = NB(d^t) - C^t(q^t, d^t, \mathbf{g}^t, \mathbf{c}^t) \tag{4.12}$$

However, if demand is assumed to be inelastic, then $NB(d^t)$ is constant and can be ignored. Thus:

$$B^t(q^t) = -C^t(q^t, d^t, \mathbf{g}^t, \mathbf{c}^t) \tag{4.13}$$

Figure 4.6 illustrates $B^t(q^t)$ for $d^t = 12\text{MW}$, which is $C^t(\bullet)$ flipped horizontally and shifted down the y-axis by the cost associated with q^t=0. In the absence of spill penalties, $B^t(q^t)$ is non-decreasing and increases at a non-increasing rate over the range of feasible release levels. See for illustrations of the form of $B^t(q^t)$ when demand is modelled using a linear curve.

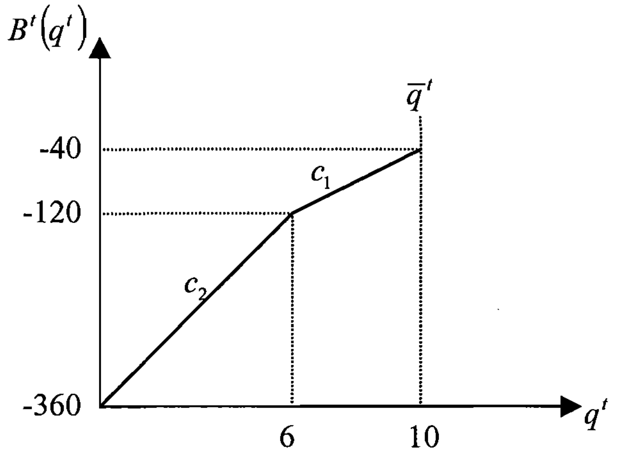


Figure 4.6: $B^t(q^t)$ for cost minimiser (fixed demand)

As discussed earlier, weekly demand can be approximated by demand curves which represent demand in each sub-period. The optimal release in each sub-period will be that which maximises $B^t(q^t)$, given some value of q^t, say q_k^t. It is assumed that w^t and s^t do not affect $B^t(q^t)$, so $B^t(q^t)$ need only be calculated once in each period. Let q_{ks}^t be the release in sub-period *s* given total release in the period is q_k^t and where *S* is the set of sub-periods. For weekly release level q_k^t, the optimal values of q_s^t maximise $B^t(q_k^t)$, or, equivalently, minimise $C^t(q_k^t, d^t, \mathbf{g}^t, \mathbf{c}^t)$ since $B^t(q_k^t) = -C^t(q_k^t, d^t, \mathbf{g}^t, \mathbf{c}^t)$. Thermal generation in each sub-period, g_{kis}^t, is also optimised since marginal costs (c_i^t) are assumed constant for each thermal station in each sub-period. Constant demand in each sub-period is denoted by d_s^t and is weighted by γ_s^t, where $\sum_{s \in S} \gamma_s^t = 1$ and γ_s^t indicates the proportion of the weekly load that d_s^t is approximating.

The optimal release allocation (**ORA**) problem can be stated as:

ORA $$\min_{q_{ks}^t, g_{kis}^t} \sum_s \sum_i g_{kis}^t c_i^t$$

subject to $$\sum_s q_{ks}^t = q_k^t$$

$$q_{ks}^t + \sum_i g_{kis}^t = \gamma_s^t d_s^t$$

$$0 \le q_{ks}^t \le \overline{q}_{ks}^t \text{ and } 0 \le g_{kis}^t \le \gamma_s^t \overline{g}_{kis}^t \tag{4.14}$$

The objective is to determine sub-period releases q_{ks}^t, and thermal generation g_{kis}^t, so as to maximise the negative (convex) costs of thermal generation in each sub-period. The first constraint ensures that the sum of the releases in each sub-period equals the total release to be allocated. Next, (weighted) demand in each sub-period is satisfied by the hydro and thermal generation. Lastly, hydro release and thermal generation are constrained to be less than the upper generation bound, which is weighted by γ_s^t for thermal plant.

In the optimal solution to **ORA** for a given q_k^t, release will be allocated to the sub-period with the largest marginal cost of thermal generation. As q_k^t is increased, and **ORA** re-solved, the additional release will be allocated to offset thermal general with the same, or lower, marginal cost. The benefit function $B^t(q^t)$ will therefore be a non-decreasing function of q^t, with a non-increasing slope.

Theorem 4.1. $B^t(q^t)$ is non-decreasing at a non-increasing rate (q^t increasing).

Proof. The optimal release allocation over the sub-periods available will offset thermal generation in order of highest to lowest marginal cost. With marginal costs non-negative, increasing q^t will cause a non-negative change in thermal generation, which will incur a non-negative cost. The associated change in $B^t(q^t)$ will therefore be non-negative and $B^t(q^t)$ will therefore be non-decreasing. An increase in q^t of one unit will reduce the total cost by the marginal cost of the most expensive operating thermal stations. With thermal costs convex, this marginal cost will decrease as q^t increases, so $B^t(q^t)$ will increase at a non-increasing rate. □

In fact, the benefit from a particular release can be calculated directly by summing the weighted sub-period RDCs to create an aggregate RDC, $RDC^t(q^t)$, by

$$RDC_A^t(q^t) = \sum_{s \in S} \gamma_s^t RDC_s^t(q^t) \tag{4.15}$$

where $RDC_s^t(q^t)$ is the RDC in a particular sub-period. The benefit from release function can then be calculated as $B^t(q^t) = -\int_{q^t}^{d_s^t} RDC_A^t(q^t)\partial q^t$. $B^t(q^t)$ could be measured in other ways, depending on the way demand is represented and the objective of the DM.

4.3.3 State transition

Given the form of $B^t(q^t)$, it is worth considering in more detail the transitions of the wealth and storage variables from period to period. Recall that the state transition equations are $w^{t+1} = w^t + B^t(q^t)$ and $s^{t+1} = s^t - q^t + a^t$ for wealth and storage, respectively. For a particular (w^t, s^t) pair, the 'state transition possibility curve' (STPC) traces the values of w^{t+1} and s^{t+1} over the range of feasible release levels.

The weekly benefit, $B^t(q^t)$, is defined here as the cost of generation supplied from other generation sources to ensure that demand is met, as introduced in the previous subsection. As was shown, $B^t(q^t)$ is calculated as the cost of (optimally dispatched) thermal generation required to meet demand. As q^t increases, the amount of thermal generation decreases, and so does the cost. A low release incurs a large cost (low benefit) and a high release incurs a low cost (high benefit). A change in the slope of the $B^t(q^t)$ corresponds to moving to an adjacent step in the stepped supply curve. Assuming these stations are dispatched in order of their marginal cost, $B^t(q^t)$ is a non-decreasing function (in release, and hence in storage) that increases at a non-increasing rate (Theorem 4.1). Each additional unit of reservoir release displaces the most expensive unit of thermal generation.

If $q^t > d^t$ the benefit remains constant since the extra release is effectively spilled. It is likely that excessive spill will be undesirable, in which case it could be penalised and the cost curve could rise (benefit fall) as the level of spill increases. In that case, precautionary spill may be desirable. In this model, q^t can not exceed d^t because $\overline{q}^t \leq d^t$ so it is not possible for spill to be optimal unless inflows imply excess storage, in which case spill can occur. This is represented in the storage balance equation by defining $s^{t+1} \le s^t - q^t + a^t$. A slack variable is used to ensure that $s^{t+1} \le \bar{s}^{t+1}$, and it will be non-zero if $s^t - q^t + a^t > \bar{s}^{t+1}$.

Figure 4.7 illustrates the STPC starting from the point (w_3^t, s_3^t), on the space $W^{t+1} \times S^{t+1}$ for a given demand d^t, and over the range of feasible release levels, $\underline{q}^t \le q^t \le \bar{q}^t$. The starting point is (w_2^t, s_2^t), point 'A', which is mapped directly on to point 'B' in $W^{t+1} \times S^{t+1}$. Note that w_3^t will not map on to w_3^{t+1} if different grid discretisations are used. The inflow is (conservatively) assumed to be zero; so that s_3^t maps to s_3^{t+1} if q^t=0, as is assumed here. The uncertain inflows are handled independently of the release decision, and hence the STPC. (In Chapter 5, it is shown how the end-of-period expected utility surface is adjusted for inflow uncertainty prior to the STPC analysis).

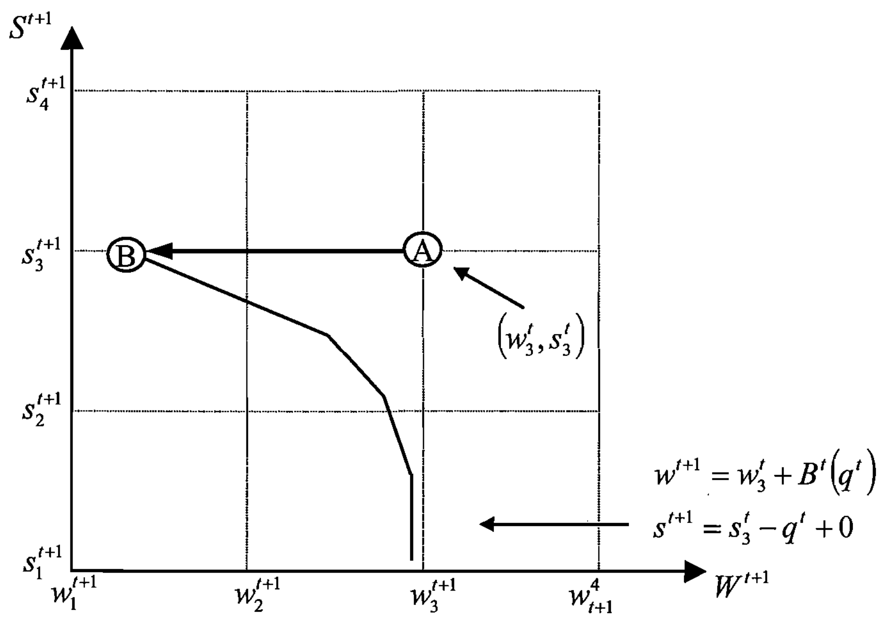


Figure 4.7: State transition possibility curve

The first point on the state transition possibility curve is point 'B', which corresponds to release $\underline{q}^t$, which in this case equals 0MWh. It is assumed here that the search starts at $\underline{q}^t$ rather than $\bar{q}^t$ or some other point. While it is not necessary to search through the release grid in any particular order, the form of the benefit function is such that the computational effort can be reduced if the search is started from either

$\underline{q}^t$ or $\overline{q}^t$. At 'B', q^t=0, so demand is met by all other supply sources, decreasing w^{t+1} to its minimum possible value because the highest possible cost is incurred (recall that the weekly benefit is the negative of the supply cost, so from a starting wealth w^t at 'A', the maximum cost is incurred by releasing $\underline{q}^t$=0). As q^t increases, s^{t+1} decreases because more water is released, and w^{t+1} increases because the additional release reduces the quantity of 'other' generation, and hence the total fuel cost. Note that as q^t increases, w^{t+1} increases at a non-increasing rate because the marginal cost of the generation displaced by the reservoir release is decreasing due to the assumption of merit order dispatch. As long as a thermal station is dispatched with some non-zero marginal cost, $w^{t+1} < w^t$.

The relationship between storage, release, and generation is assumed to be linear, so the state transition possibility curve (STPC), which is a function of wealth and storage, is independent of the value of s^t (and w^t). Head affects, which reflect the impact of the storage level on electricity generation would invalidate this assumption since increased storage generally increases the rate at which throughput can be converted to electricity. In more detailed river chain modelling, concave efficiency curves are also used to reflect the ability of turbines to convert water throughput into generation. Because the 'reservoir' in this model is actually the aggregation of two river chain systems, explicit modelling of head effects and efficiency curves would be difficult, though still possible. Another extension involves tributary inflows, which can not be stored and hence are 'lost' if not used. If the tributary flow in period t is denoted by tr^t, then tr^t would have the effect of altering wealth but not storage for $q^t \le tr^t$, shifting the STPC to the right by $B^t(tr^t)$ for $q^t \le tr^t$. Again, this is relatively easy to implement but for the cases discussed here tr^t=0 (as in Scott (1997)).

4.4 Utility functions

A utility function can be used to evaluate decisions with uncertain consequences where the 'best' decision is that which maximises expected utility. Conditions under which expected utility is a suitable measure for comparing uncertain consequences were proposed by Von Neumann and Morgenstern (1947). These axioms have been criticised and shown not to hold empirically as a descriptive model of behaviour (e.g, Kahneman and Tversky, 1979), but still remain a useful starting point for decision making under uncertainty (Bell, 1995).

Consider again the **RM-R** model, where the decisions made throughout the planning had an associated return and also affected the state of the system at the end of the planning horizon. At the end of the horizon, the consequences of the sequence of decisions (and uncertainty) are the accumulated returns and the state of the system. These attributes are used to measure the achievement of the (two) implicit objectives of the problem. In order to link decision analysis with SDP, it is assumed that the objectives of the DM relate to the expected returns and the system state (for the case of reservoir management discussed here, the system state is the reservoir storage level). Given that these attributes have been defined, the focus is on how preferences for the outcomes of those attributes affect decisions made throughout the planning horizon.

The terminal value function in **RM-R** is $f^{T+1}\left(w^{T+1}, s^{T+1}\right) = U\left(w^{T+1}, s^{T+1}\right)$. This utility function is defined over the two 'attributes', w^{T+1} and s^{T+1}, where an attribute measures the performance of an objective. For the reservoir management problem, a non-decreasing von Neumann-Morgenstern utility function would combine the consequences (w^{T+1} and s^{T+1}) into a scalar such that

$$\mathrm{E}\left[U\left(w_1^{T+1}, s_1^{T+1}\right)\right] > \mathrm{E}\left[U\left(w_2^{T+1}, s_2^{T+1}\right)\right] \Leftrightarrow \left(w_1^{T+1}, s_1^{T+1}\right) \succ \left(w_2^{T+1}, s_2^{T+1}\right) \tag{4.16}$$

and

$$\mathrm{E}\left[U\left(w_1^{T+1}, s_1^{T+1}\right)\right] = \mathrm{E}\left[U\left(w_2^{T+1}, s_2^{T+1}\right)\right] \Leftrightarrow \left(w_1^{T+1}, s_1^{T+1}\right) \sim \left(w_2^{T+1}, s_2^{T+1}\right) \tag{4.17}$$

where the $\succ$ means "preferred to" and $\sim$ means "indifferent to" (Keeney and Raiffa, 1976).

Keeney and Raiffa (1976) describe forms of multi-attribute utility functions and how they can be can be assessed given a variety of assumptions about the interaction between the DM's preferences for the attributes. The assessment of utility functions is not a prime concern here. What is of interest, though, is the nature of the DM's preferences implied by the utility functions. If plausible forms can be identified, then SUMDP can be used to assess the implications of such utility functions on system performance. Alternatively, SUMDP can be solved for a variety of plausible utility functions and the DM can select a preferred policy based on its performance. Where multiple DM's are involved, this approach could be used to facilitate discussion and quantification of risk because the impact of different utility functions on, say, the distributions of end-of horizon storage and accumulated returns, can be assessed.

One approach for modelling preferences for multiple attributes, and the one used here, is to define an additive utility function (see for example, Keeney, 1970; Richard, 1975). For the reservoir management problem described earlier, this would have the form

$$U\left(w^{T+1}, s^{T+1}\right) = k_w u_w\left(w^{T+1}\right) + k_w u_s\left(s^{T+1}\right) \tag{4.18}$$

Scalars k_w and k_s can be used to weight the utility functions. A function with this form implies that w^{T+1} and s^{T+1} are mutually utility independent, which means that the utility associated with each consequence is independent of the value of the other consequence.

In a practical setting, a DM's utility function (perhaps derived using the methods in Keeney and Raiffa (1976)) may well have a different form. The additive utility function provides a useful starting point, though. Furthermore, from a modelling and experimental perspective, it is easy to adjust the independent preferences for storage and wealth because only the particular utility function needs to be altered.

4.4.1 Risk attitudes

Before considering the risk attitudes implied by a multi-attribute utility function, as discussed above, first consider the case for a single attribute. Risk attitudes can be illustrated for the case of a DM deciding between two alternatives with the consequences measured using a single attribute (say returns). The DM's utility is assumed to be non-decreasing in returns, so more is better. One alternative, $\overline{w}$, has a certain return and the other, $\widetilde{w}$ has uncertain returns and is 'risky'. Utility maximisation means that the If $\overline{w}$ is preferred to $\widetilde{w}$, and $\overline{x} = E[\widetilde{w}]$, then

$$U(\mathrm{E}[\widetilde{w}]) > \mathrm{E}[U(\widetilde{w})] \tag{4.19}$$

and the DM is said to be risk averse if this holds over all possible $\widetilde{w}$, and implies that the DM's utility function is concave. On the other hand, if $\widetilde{x}$ is preferred to $\overline{x}$, the DM is said to be risk prone, and the utility function is convex. The DM is said to be risk neutral if the DM is indifferent between the certain and risky alternatives, such that $U(\mathrm{E}[\tilde{w}]) = \mathrm{E}[U(\tilde{w})]$. These relationships hold given an initial level of wealth (or asset position) w_0, so the DM is risk seeking, for example, if $U(\mathrm{E}[w_0 + \tilde{w}]) < \mathrm{E}[U(w_0 + \tilde{w})]$. In terms of the functional form of *U*, $\partial U / d^2 w \leq 0$ means *U* is concave and hence the DM is risk averse. *U* is convex and the DM risk seeking if $\partial U / d^2 w \geq 0$.

Richard (1975) extends the results from single attribute utility functions to multi-attribute (or multivariate) utility functions. The DM's utility is evaluated over two uncertain outcomes, say $\tilde{w}$ and $\tilde{s}$. Briefly, consider the DM's preferences for the following alternatives, each with equally probable outcomes: $A = ((w_0, s_0), (w_1, s_1))$ and $B = ((w_0, s_1), (w_1, s_0))$, where $w_1 > w_0$ and $s_1 > s_0$. The DM is

- multi-attribute risk averse if $B \succ A$ for all w_0, w_1, s_0, and s_1;
- multi-attribute risk neutral if and only if $A \sim B$ for all w_0, w_1, s_0, and s_1; and
- multi-attribute risk seeking if $A \succ B$.

When examining the effect of different utility functions on system performance, having a measure of the degree of risk aversion implied by a utility function is useful, but not necessary. Unfortunately, there are no simple measures of risk aversion for multiple attributes, though it is possible to indicate whether a utility function implies greater risk aversion than another i.e., an ordinal measure rather than a cardinal one (Richard, 1975). Essentially, the multi-attribute risk premiums can be compared for two different utility functions, and that with the larger premium is more risk averse. See for further detail.

4.4.2 A special case: w^{T+1} and s^{T+1} exchangeable

It is possible to define $U(w^{T+1}, s^{T+1})$ such that auxiliary variable w^{T+1} is not required, even though the consequence (w^{T+1}, s^{T+1}) is uncertain. This is possible if utility is effectively defined as a function of the sum of w^{T+1} and s^{T+1}, or more generally if:

$$U(w^{T+1}, s^{T+1}) = u(w^{T+1} + V(s^{T+1})) \tag{4.20}$$

The value of the terminal state of the system has been transformed into the units of w^{T+1} using $V(s^{T+1})$. With this definition, the DM's utility is dependent on the aggregate 'value' associated with the consequence (w^{T+1}, s^{T+1}), so the values of the system state and wealth are implicitly exchangeable i.e.,

$$\frac{\partial u(w^{T+1}, s^{T+1})}{\partial w^{T+1}} = \frac{\partial u(w^{T+1}, s^{T+1})}{\partial V(s_{T+1})} \tag{4.21}$$

If u is linear this definition of $U(w^{T+1}, s^{T+1})$, then the problem is equivalent to **P1** because the value function is simply a linear transformation of that in **P1** and the level of accumulated wealth no longer affects the marginal benefit from the release decision i.e., the release decision is independent of w^T, and only depends on s^T and the trade off between the immediate return in t and the expected future value. This can be illustrated by considering the decision made in period T. If the contours of $U(w^{T+1}, s^{T+1})$ were plotted on $W^{T+1} \times S^{T+1}$, they would have the form illustrated in Figure 4.8.

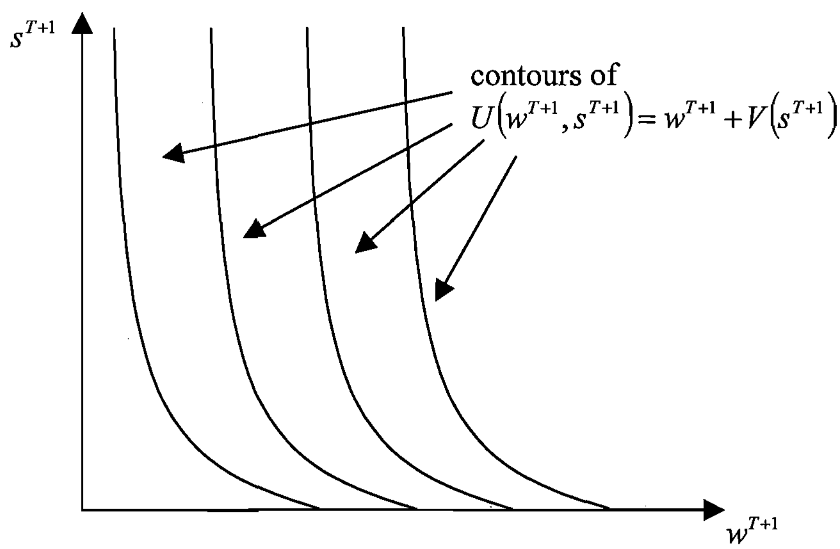


Figure 4.8: $U(w^{T+1}, s^{T+1}) = u(w^{T+1} + V(s^{T+1}))$ **with** u **linear**

Along each contour, $U(w^{T+1}, s^{T+1})$ is constant. The contours are curved because $V'(s^{T+1})$ is non-increasing (as in **P1**). They are equally spaced because increasing w^{T+1} results in a linear increase in $U(w^{T+1}, s^{T+1})$. Importantly, the contours are identical in shape as w^{T+1} varies, so the marginal value of storage in T+1 only depends on the value of s^{T+1}. Therefore, the release decision that is optimal for a particular value of s^T will be optimal for all w^{T+1} and involves equating the marginal utility of release in *T* with the marginal utility of storage in *T+1*. This logic applies for decisions in all $t \in T$.

What is interesting is that the same result occurs if *u* is non-decreasing *and* non-linear *and* the uncertainty in *t* (in this case due to uncertainty about inflow a^t) only affects the system state in the subsequent period, s^{t+1}. The reason is that the contours (or indifference curves) of $U\left(w^{T+1}, s^{T+1}\right)$ remain as linear transformations of each other. While non-linear *u* will result in contours that are not evenly spaced in the w^{T+1} dimension (as in Figure 4.8), the form of the contours does remain the same. The slope of a contour at a particular value of s^{T+1} is the same for any value of w^{T+1}, so

$$u\left(w_1^{T+1}, s_1^{T+1}\right) = u\left(w_1^{T+1} - \left[V\left(s_2^{T+1} - s_1^{T+1}\right)\right], s_2^{T+1}\right) \tag{4.22}$$

and the marginal rate of substitution between w^{T+1} and s^{T+1} depends only on s^{T+1} and not on w^{T+1} (Keeney and Raiffa, 1976). The optimal decision in *T* will therefore only be a function of s^{T+1}.

The mathematical derivation of this result is as follows. Assume that *u* is continuous and differentiable. A contour of *u* describes the relationship between *w* and *s* for a fixed value of $U(\bullet)$, say *k* (the *T+1* subscripts have been dropped from *w* and *s*). At a given point $\left(w_1^{T+1}, s_1^{T+1}\right)$, the marginal rate of substitution (MRS) reflects the change in one variable required to compensate for a change in the other to maintain the same value of $U\left(w^{T+1}, s^{T+1}\right)$; it is the slope of the contour. The MRS at $\left(w_1^{T+1}, s_1^{T+1}\right)$ for $U\left(w^{T+1}, s^{T+1}\right)=k$ is described as

$$k = -\frac{\partial w}{\partial s}\bigg|_{x_1, s_1} = \frac{\partial u(x_1, s_1)}{\partial s} \bigg/ \frac{\partial u(x_1, s_1)}{\partial w} \tag{4.23}$$

With *s* 'valued' using $V(s)$, $\dfrac{\partial u(x_1, s_1)}{\partial s}$ is

$$\frac{\partial u(w_1, s_1)}{\partial s} = \frac{\partial u(w_1, s_1)}{\partial V(s)} \frac{\partial V(s)}{\partial s} \tag{4.24}$$

It was shown in Equation 4.21 that $u\left(w^{T+1}+V\left(s^{T+1}\right)\right)$ implies $\frac{\partial u}{\partial w}=\frac{\partial u}{\partial V(s)}$. Substituting Equation 4.21 in to the definition of $\frac{\partial u}{\partial s}$ gives

$$\frac{\partial u(w_1,s_1)}{\partial s}=\frac{\partial u(w_1,s_1)}{\partial w}\frac{\partial V(s)}{\partial s} \tag{4.25}$$

Substituting this into the original contour definition gives

$$\begin{aligned}\left.\frac{\partial w}{\partial s}\right|_{w_1,s_1} &= \left.\frac{\partial u(w_1,s_1)}{\partial w}\frac{\partial V(s)}{\partial s}\right/\frac{\partial u(w_1,s_1)}{\partial w} \\ &= \frac{\partial V(s)}{\partial s}\end{aligned} \tag{4.26}$$

The MRS for $\left(w_1^{T+1},s_1^{T+1}\right)$ is therefore independent of the value of w_1^{T+1}, and this will apply to all $w^{T+1}\in W^{T+1}$. Illustrative contours of $U\left(w^{T+1},s^{T+1}\right)$ are plotted in Figure 4.9, they would be horizontal translations of each other and parallel in the wealth dimension.

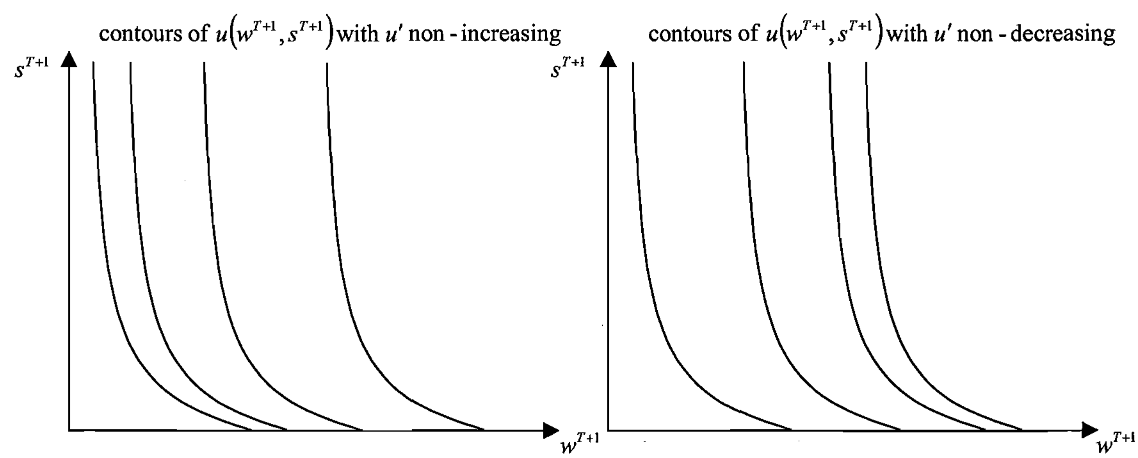


Figure 4.9: $U\left(w^{T+1},s^{T+1}\right)=u\left(w^{T+1}+V\left(s^{T+1}\right)\right)$ **with *u* non-decreasing**

4.5 Conclusions

The conventional approach taken in reservoir management models involves minimising the expected cost of operation. While 'risk' has been identified as an important aspect of reservoir management, there are actually very few models which explicitly consider risk. In this chapter, SUMDP has been presented and discussed as an approach to reservoir management in a 'regulated' planning environment, which corresponds to the classical 'cost minimiser' scenario frequently considered in the literature. As in classical DP models, the return (or benefit) from release in each period is simply the cost of generation that is required to meet demand. As in classical decision theory, a utility function is defined over the appropriate consequences of decisions, which in this case are the storage level and accumulated returns at the end of the planning horizon. Combining these two concepts can be problematic, though, because utility maximisation generally invalidates a traditional DP approach due to the non-separability of the objective. SUMDP overcomes this by augmenting the state space of a conventional DP model with an accumulated 'wealth' variable. As a result, the scope of utility functions able to be considered for reservoir management problems is not restricted to linear utility curves.

In Chapter 5, implementation issues associated with solving the regulated reservoir management (**RM-R**) model using discrete stochastic dynamic programming are discussed. Chapter 6 contains a discussion of optimisation and simulation results for **RM-R** applied to a reservoir management problem using a representation of the New Zealand electricity system.

Chapter 5

Implementation Issues

5.1 Introduction

In Chapter 4, a SUMDP approach to reservoir management in a regulated environment was discussed (the **RM-R** model). In this chapter, issues relating to solving the recursive relation in each period are discussed. The solution approach is discussed and illustrated in Section 5.2 and discretisation issues are discussed in Section 5.3. As a consequence of the form of the benefit and utility function, the solution algorithm can be modified to reduce the computation time, as discussed in Section 5.4.

5.2 Solution approach

The **RM-R** model can be solved using discrete dynamic programming. This involves discretising the continuous variables and the cost-to-go function evaluated at combinations of the discrete values (Denardo, 1982). The value function is therefore approximated at these discrete combinations, and the accuracy of the approximation is contingent on the resolution of the discretisation.

A basic solution algorithm for solving **RM-R** is shown in Figure 5.1.

Determine $f^{T+1}\left(w_m^T, s_n^T\right)$
For $t = T \ldots 1$ (period)
  For $m = 1 \ldots M$ (wealth w_m^t)
    For $n = 1 \ldots N$ (storage s_n^t)
      $f^t\left(w_m^t, s_n^t\right) = -\infty$
      For $k = 1 \ldots K$ given $q_k^t < s_n^t$ (release q_k^t)
        For $l = 1 \ldots L$ (inflow a_l^t)
          $w^{t+1} = w_m^t + B^t\left(q_k^t\right)$
          $s^{t+1} = s_n^t - q_k^t + a_l^t$
          $\bar{g}^t\left(w^{t+1}, s^{t+1}\right) = f^{t+1}\left(w^{t+1}, s^{t+1}\right)$
        end for l
        $\bar{f}_{m,n,k}^t = \sum_l p_l^t \bar{g}_l^{t+1}\left(w^{t+1}, s^{t+1}\right)$
        If $\bar{f}_{m,n,k}^t > f^t\left(w_m^t, s_n^t\right)$
          $f^t\left(w_m^t, s_n^t\right) = \bar{f}_{m,n,k}^t$
          $\hat{q}^t\left(w_m^t, s_n^t\right) = q_k^t$
        end if
      end for k
    end for n
  end for m
end for t

Figure 5.1: Basic SDP solution algorithm

Given the terminal value function i.e., the utility function defined over w^{T+1} wealth and storage, the terminal cost-to-go function can be determined for the range of storage and wealth values. The first period considered by the algorithm is period *T*, and the decisions made in *T* will determine the (probability distribution of) values of wealth and storage at the end of *T*/beginning of *T+1* over which the terminal value function is defined. Due to the discretisation of the variables, there are a number of loops in the algorithm. Once a discrete level of wealth (index *m*) and storage (index *n*) have been selected, the expected cost to go is determined for each feasible discrete release level (index *k*).

Inflows are the stochastic element of this reservoir planning problem. In each week, the inflow (a^t) into the reservoir is unknown. However, the distribution of these inflows is assumed to be known in advance, and is also assumed to be independent of the level of inflow in any other period. The inflow distribution in each week is sampled at, or represented by, L values, each with some finite probability of occurring. Let a_l^t denote the value of sample l and $p_l^t = \Pr\left[a^t = a_l^t\right]$. Recall that inflows are assumed to occur after the release decision has been made and the state transition for storage is $s^{t+1} \leq s^t - q^t + a^t$.

5.3 Discretisation issues

The discretisations of storage and release are likely to remain constant over the planning horizon due to the bounds on those variables being constant. However, for the wealth variable as it is defined here, $B^t\left(q^t\right)$ is accumulated over the planning horizon and the range of possible w^t values is very large compared to $B^t\left(q^t\right)$. If the number of grid points is kept constant, then the change in utility corresponding to relative changes in w^t will be covered by a small number of grid points, and the approximation may be coarser than desired.

In terms of defining the grid points in this model, the discretisation scheme for wealth does not involve discretising evenly over the entire range of possible wealth values in each period, i.e., from $\underline{W}^t$ to $\overline{W}^t$ in t. The reason for this is that it is unlikely that release will be consistently low (or high) throughout the entire planning horizon. Note that $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ is defined over the range of w^{T+1} (and s^{T+1}). It is not necessary to discretise the w^{T+1} variable if $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ is defined in functional form since the value of $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ for any value of w^{T+1} and s^{T+1} can be computed directly. Thus, the following discussion of discretisation of w^t relates to periods 1…T, with T being the last period in which discretisation occurs. Were discretisation to be required in $T+1$, then the approach discussed below remains the same with the reference to T changed to T+1.

To improve the 'focus' of the state space discretisation, bounds on the w^t variables were set manually. The upper and lower bounds in T are denoted by the variables $\overline{W}_H^T$ and $\underline{W}_H^T$, respectively. These bounds define a smaller 'focus region' within which the discretisation is evenly spaced ($\overline{W}_H^T \leq w^T \leq \overline{W}_H^T$), as illustrated in Figure 5.2. Calculations for $\underline{W}^T \leq w^T \leq \underline{W}_H^T$ and $\overline{W}_H^T \leq w^T \leq \overline{W}^T$ could still be performed, but interpolations in this extended range were made over a much coarser grid (in fact the regions were only discretised at the two end points that defined them).

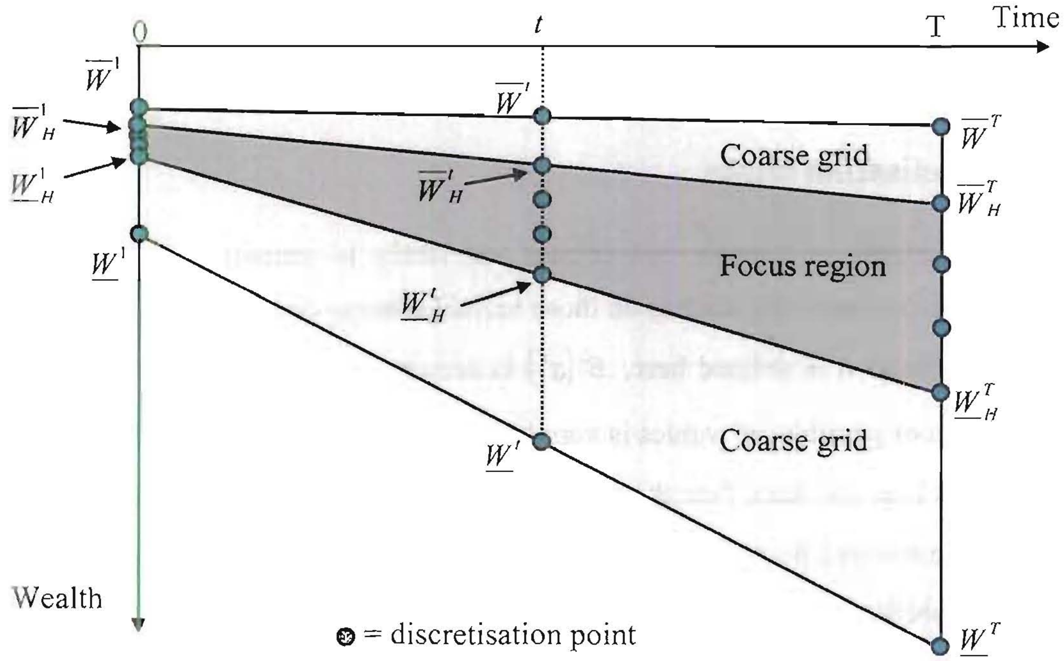


Figure 5.2: Discretisation of wealth variable

Bounds were also set on the grid points in the first period. For each week, $\underline{W}_H^t$ was calculated by interpolating along the line defined by $\underline{W}_H^1$ and $\underline{W}_H^T$. Similarly, $\overline{W}_H^t$ was calculated by interpolating between $\overline{W}_H^1$ and $\overline{W}_H^T$. For a given week, then, the wealth grid between $\overline{W}_H^t$ and $\underline{W}_H^t$ was divided into M-3 equally spaced grid points, where M is the number of discrete values used to approximate w^t. A 'constant grid' discretisation, whereby the distance between any two grid points is always the same in any period, could be used in conjunction with, or as an alternative to, the approach described here. However, the concept of the 'focus region' remains valid, so it would be logical to apply this discretisation only to wealth values in the 'focus region'.

Given the discretisation of the wealth/storage space in a given period, the value of $f^t(w^t, s^t)$ is evaluated for the feasible range of releases and the maximum value (and associated release) stored. The conventional approach (as in Figure 5.1) for this process is to calculate a candidate $f^t(w^t, s^t)$ as the weighted sum of the $f^{t+1}(w^{t+1}, s^{t+1})$ values corresponding to each of the possible ending states which result from the possible inflows (see for example Nemhauser (1966) or Denardo (1982)). The probability of the ending state corresponds to the probability of the inflow used to derive it (p_l^t). If $\bar{f}_{m,n,k}^t$ denotes the expected utility associated with wealth w_m^t, storage s_n^t, and release q_k^t, all in *t*, then

$$\bar{f}_{m,n,k}^t = \sum_l p_l^t f^{t+1}\left(w_m^t + B^t\left(q_k^t\right), s_n^t - q_k^t + a_l^t\right) \tag{5.1}$$

Let $\bar{g}^t\left(w^{t+1}, s^{t+1}\right) = f^{t+1}\left(w_m^t + B^t\left(q_k^t\right), s_n^t - q_k^t + a_l^t\right)$ and be determined by interpolating between the values of $\bar{g}^t\left(w^{t+1}, s^{t+1}\right)$ at the grid points representing combinations of w_x^{t+1}, w_{x+1}^{t+1}, s_y^{t+1}, and s_{y+1}^{t+1}, where $w_x^{t+1} \le w^{t+1} \le w_{x+1}^{t+1}$ and $s_y^{t+1} \le s^{t+1} \le s_{y+1}^{t+1}$ (see Hadley (1964)). A variety of approximation schemes exist. The approximation scheme used here is detailed in . For a fixed discretisation of release, $\bar{g}^t\left(w^{t+1}, s^{t+1}\right)$ is calculated a maximum of *L* times for each q_k^t for a given (w_m^t, s_n^t). The algorithm in Figure 5.1 will therefore require that $\bar{g}^t\left(w^{t+1}, s^{t+1}\right)$ be calculated $M \times N \times K \times L$ times in each period. The computational requirements can be reduced, though, as will be shown in later sections.

5.4 Computational improvements

The computational effort required to determine $f^t\left(w^t, s^t\right)$ and $\hat{q}^t\left(w_m^t, s_n^t\right)$ in each period can be reduced by utilising the form of, and interaction between, the benefit function and the expected end-of-period utility function. This dampens the effect of increasing the dimensionality of the problem by including the wealth state variable. These refinements are discussed in the following sub-sections, and modifications are proposed to the SDP algorithm presented earlier in Figure 5.1.

As discussed earlier, $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ is defined as $U\left(w^{T+1}, s^{T+1}\right) = k_w u_w\left(w^{T+1}\right) + k_w u_s\left(s^{T+1}\right)$. Results discussed later will assume that $u_w\left(w^{T+1}\right)$ and $u_s\left(s^{T+1}\right)$ are both non-decreasing at a non-increasing rate. This in turn implies that $U\left(w^{T+1}, s^{T+1}\right)$ and hence $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ are non-decreasing at a non-increasing rate.

Assumption 1: $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ is non-decreasing at a non-increasing rate.

Although the values of $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ are only known at discrete points in the wealth/storage space, it is convenient to assume that $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ is continuous and twice differentiable, so that

$$\frac{\partial f^{T+1}\left(w^{T+1}, s^{T+1}\right)}{\partial w^{T+1}} \geq 0 \text{ and } \frac{\partial^2 f^{T+1}\left(w^{T+1}, s^{T+1}\right)}{\partial\left(w^{T+1}\right)^2} \leq 0 \tag{5.2}$$

Similarly for storage,

$$\frac{\partial f^{T+1}\left(w^{T+1}, s^{T+1}\right)}{\partial s^{T+1}} \geq 0 \text{ and } \frac{\partial^2 f^{T+1}\left(w^{T+1}, s^{T+1}\right)}{\partial\left(s^{T+1}\right)^2} \leq 0 \tag{5.3}$$

Using discrete dynamic programming means that $f^t\left(w^t, s^t\right)$ is not continuous, but is approximated at the wealth/storage grid points in t. Therefore, it is assumed that the first and second forward differences, which are the discrete analogues of the first and second derivatives, are positive and negative respectively (Denardo (1982)).

5.4.1 Inflow uncertainty adjustment

The approach to handling inflows presented in the basic SDP solution algorithm (Figure 5.1) is inefficient because $f^{t+1}\left(w_m^t + B^t\left(q_k^t\right), s_n^t - q_k^t + a_l^t\right)$ may be calculated several times during the search of the state space. Because a_l^t is independent of the values of wealth and release, the end-of-horizon expected utility function can be adjusted for inflows independently of the release decision, meaning that the adjustment for inflow uncertainty need only be performed once and reducing the computational requirements accordingly. This concept is not unique and has been used in other reservoir management applications (see for example Scott (1997) or Scott and Read (1996)).

The following discussion is presented for a generic period t, since $f^{t+1}\left(w^{t+1}, s^{t+1}\right)$ is only assumed to be a non-decreasing function of w^{t+1} and s^{t+1}, which is less restrictive than the definition of $f^{t+1}\left(w^{t+1}, s^{t+1}\right)$ in Assumption 1.

Let $g^t\left(w^{t+1}, s^{t+1}\right)$ be the 'inflow adjusted' value surface, where

$$g^t\left(w_m^{t+1}, s_n^{t+1}\right) = \sum_l p_l^t f^{t+1}\left(w_m^{t+1}, s_n^{t+1} + a_l^t\right) \qquad \forall m,n \qquad (5.4)$$

Here, $g^t\left(w^{t+1}, s^{t+1}\right)$ is just a convex combination of points in $f^{t+1}\left(w^{t+1}, s^{t+1}\right)$, so the properties of $f^{t+1}\left(w^{t+1}, s^{t+1}\right)$ will be evident in $g^t\left(w^{t+1}, s^{t+1}\right)$.

Theorem 5.1. *The function $g^t\left(w^{t+1}, s^{t+1}\right)$ will be non-decreasing at a non-increasing rate after performing the inflow uncertainty adjustment as in Eqn 5.4.*

Proof. The inflow distribution is identical for all values of storage and wealth. The inflow adjustment for any pair of discrete $\left(w^{t+1}, s^{t+1}\right)$ therefore corresponds to creating a convex combination of $f^{t+1}\left(w^{t+1}, s^{t+1}\right)$ values, where $\bar{s}^{t+1} \geq s^{t+1}$. If $f^{t+1}\left(w^{t+1}, s^{t+1}\right)$ is non-decreasing (non-increasing), a convex combination of $f^{t+1}\left(w^{t+1}, s^{t+1}\right)$ will yield another non-decreasing (non-increasing) function. □

The expected utility associated with different $\left(w^{t+1}, s^{t+1}\right)$ can then be determined by reading/interpolating the value of $g^t\left(w^{t+1}, s^{t+1}\right)$ associated with $\left(w^{t+1}, s^{t+1}\right)$; only a single interpolation is required for each discrete release level. Thus, $\bar{f}_{m,n,k}^t$ can be redefined as

$$\bar{f}_{m,n,k}^t = g^t\left(w_m^t + B^t\left(q_k^t\right), s_n^t - q_k^t\right) \qquad (5.5)$$

As this is a function representing the expected utility prior to the inflow at the end of t, and the release at the start of t, the subscript on the 'g' is t. Because the benefit from release is not accounted for, $g^t\left(w^{t+1}, s^{t+1}\right)$ is still defined over the values of wealth which occur at the beginning of the next period. The SDP algorithm with this inflow uncertainty adjustment is shown in Figure 5.3.

Determine $f^{T+1}\left(w_m^T, s_n^T\right)$
For $m=1\ldots M$ (wealth w_m^{t+1})
  For $n=1\ldots N$ (storage s_n^{t+1})
    For $l=1\ldots L$ (inflow a_l^t)
      $g_l^t = f^{t+1}\left(w^{t+1}, \min\left(\bar{S}^t, s_n^t + a_l^t\right)\right)$
    end for l
    $g^t\left(w^{t+1}, s^{t+1}\right) = \sum_l p_l^t g_l^t$
  end for n
end for m

For $t=T\ldots 1$ (period)
  For $m=1\ldots M$ (wealth w_m^t)
    For $n=1\ldots N$ (storage s_n^t)
      $f^t\left(w_m^t, s_n^t\right) = -\infty$
      For $k=1\ldots K$ given $q_k^t < s_n^t$ (release q_k^t)
        $w^{t+1} = w_m^t + B^t\left(q_k^t\right)$
        $s^{t+1} = s_n^t - q_k^t$
        $f_{m,n,k}^t = g^t\left(w^{t+1}, s^{t+1}\right)$
        If $\bar{f}_{m,n,k}^t > f^t\left(w_m^t, s_n^t\right)$
          $f^t\left(w_m^t, s_n^t\right) = \bar{f}_{m,n,k}^t$
          $\hat{q}^t\left(w_m^t, s_n^t\right) = q_k^t$
        end if
      end for k
    end for n
  end for m
end for t

Figure 5.3: Solution algorithm with separate inflow uncertainty adjustment

Without the inflow adjustment, a maximum of $M \times N \times K \times L$ interpolations are required (this is the worst case). The 'inflow adjustment' procedure requires $M \times N \times L$ interpolations, while the 'release decision' requires $M \times N \times K$ interpolations. In the absence of any other modifications, then, the total number of interpolations required to derive $f^t\left(w^t, s^t\right)$ and is $M \times N \times (K+L)$. Therefore, calculating $g^t\left(w^{t+1}, s^{t+1}\right)$ decreases the number of interpolations required by a factor of $\left.(K+L)\middle/KL\right. \cong \frac{1}{L}$ per period (ignoring other overheads). More efficient methods may yield even larger savings (see Scott (1997a) for example). For the experiments detailed in this thesis, inflows were assumed to be Normally distributed and sampled at 5 points (L=5) with probabilities 0.15, 0.2, 0.3, 0.2, and 0.15.

The savings from adjusting using $f^t(w^t, s^t)$ for inflows are particularly valuable because they are independent of the form of the utility functions and the benefit curve. For example, consider the case of demand uncertainty, where the demand in each period is approximated with discrete outcomes and probabilities. A single release will then correspond to several values of $B^t(q^t)$, so the expectation must be taken over the corresponding values of w^{t+1} (ignoring storage). Another release level could imply the same w^{t+1} from a different w^t, but, without very restrictive assumptions, the associated values of $B^t(q^t)$ can not be guaranteed to be the same.

On the other hand, it is not difficult to imagine uncertain variables which would allow a 'wealth adjustment' to be made. For example, in a regulated market the contract price could be modelled as a stochastic variable. With a fixed contract level, the contract revenue would be uncertain and independent of the level of storage and release. In the context of the stochastic route choice problem (), it is not unreasonable to imagine a project management, or capacity expansion problem, where some sort of cash injection is expected at the completion of some set of tasks, but that the value of that injection is uncertain. Whether or not a separate adjustment for uncertain wealth improves the tractability of the algorithm depends on the nature of the uncertainty.

5.4.2 Release ratchet

The following discussion shows how the search for the optimal release in T for a particular wealth/storage pair does not require $g^T(w^{T+1}, s^{T+1})$ be evaluated for all release levels. The 'release ratchet' approach described here is so named because instead of starting the search for the optimum release level from the minimum feasible release for every instance of the discrete state space, a 'ratchet' release level can be defined as the starting point for the search. This 'ratchet' release level is updated as the state space is evaluated and only moves in one direction, so the set of release levels to be evaluated continually decreases. Read (1986) used the same concept in his algorithm for deriving optimal oil stockpile capacity.

From Theorem 5.1, $g^T\left(w^{T+1}, s^{T+1}\right)$, which is $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ adjusted for the stochastic inflows in *T*, will have the same properties as $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ i.e.,

$$\frac{\partial g^T\left(w^{T+1}, s^{T+1}\right)}{\partial w^{T+1}} \geq 0 \text{ and } \frac{\partial^2 g^T\left(w^{T+1}, s^{T+1}\right)}{\partial\left(w^{T+1}\right)^2} \leq 0 \tag{5.6}$$

Similarly for storage,

$$\frac{\partial g^T\left(w^{T+1}, s^{T+1}\right)}{\partial s^{T+1}} \geq 0 \text{ and } \frac{\partial^2 g^T\left(w^{T+1}, s^{T+1}\right)}{\partial\left(s^{T+1}\right)^2} \leq 0 \tag{5.1}$$

For convenience, these functions are treated as continuous here, though will be discretised in the model.

The slope of a contour of $g^T\left(w^{T+1}, s^{T+1}\right)$, which has some constant value of expected end-of-horizon utility, is defined as

$$\left. \frac{\partial g^T\left(w^{T+1}, s^{T+1}\right)}{\partial s^{T+1}} \middle/ \frac{\partial g^T\left(w^{T+1}, s^{T+1}\right)}{\partial w^{T+1}} \right. = \frac{\partial w^{T+1}}{\partial s^{T+1}} \tag{5.7}$$

An example of the form of the contours of $g^T\left(w^{T+1}, s^{T+1}\right)$ is shown in Figure 5.4. With $g^T\left(w^{T+1}, s^{T+1}\right)$ non-decreasing, the value of expected utility associated with each contour increases from left to right, from low wealth/storage to high wealth/storage. The impact of $g^T\left(w^{T+1}, s^{T+1}\right)$ having a non-increasing slope is that contours are spaced further apart in the wealth/storage space, with the increased distance between contours indicating lower marginal utility.

Consider one of the contours in Figure 5.4. Starting from a high wealth/low storage position, if wealth is decreased, utility decreases, so storage must be increased to maintain the same level of utility (i.e., moving up the contour). Because the marginal utility of storage decreases, the increase in storage required to maintain the level of utility increases as wealth is reduced, which results in each contour having a steeper slope as wealth decreases and storage increases. Conversely, starting from a high storage/low wealth position on a contour and decreasing storage requires that wealth be increased to maintain the same value of utility, and at an increasing rate. Hence each contour becomes flatter as storage decreases and wealth increases.

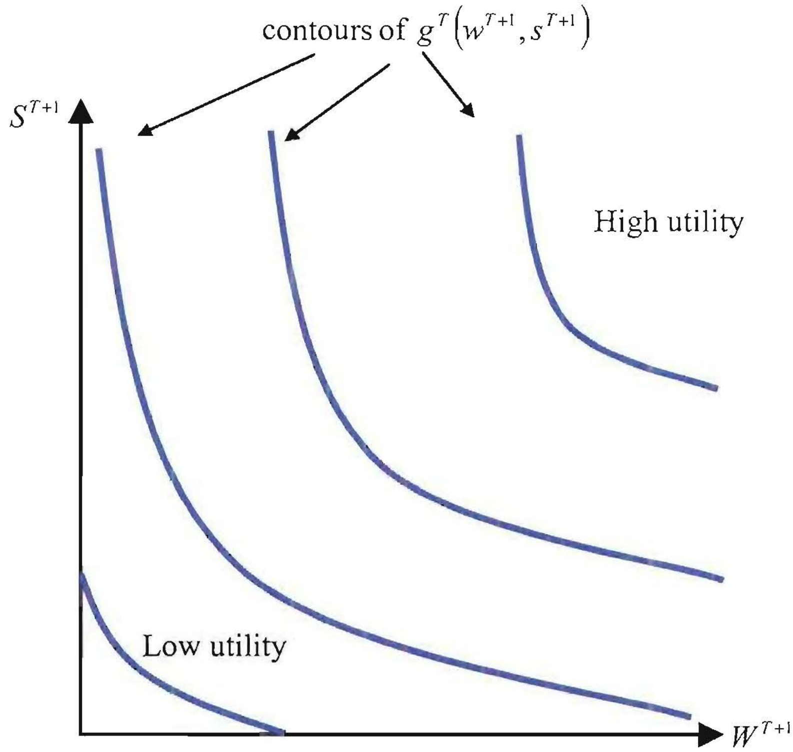


Figure 5.4: Contours of the utility function

The value of $g^T(w^{T+1}, s^{T+1})$ associated with any combination of wealth and storage will not be less for the pair with the higher storage level when two pairs have equal values of wealth, and vice versa with storage fixed. That is, for (w_m^T, s_n^T), (w_m^T, s_{n+1}^T), (w_{m+1}^T, s_{n+1}^T) with $s_{n+1}^T \geq s_n^T$ and $w_{m+1}^T \geq w_m^T$, $g^T(w_m^T, s_{n+1}^T) \geq g^T(w_m^T, s_n^T)$, $g^T(w_{m+1}^T, s_n^T) \geq g^T(w_m^T, s_n^T)$, and $g^T(w_{m+1}^T, s_{n+1}^T) \geq \max(g^T(w_m^T, s_{n+1}^T), g^T(w_{m+1}^T, s_n^T))$. Consider now the contours of $g^T(w^{T+1}, s^{T+1})$ in Figure 5.4 with w^T and s^T on the *x* and *y* axes, respectively. Marginal utility decreases as storage and wealth increase. Any contour passing through a vertical line (fixed wealth) must have a greater slope at (w_m^T, s_{n+1}^T) than at (w_m^T, s_n^T), for any $s_{n+1}^T \geq s_n^T$ i.e,

$$\partial g^T(w_m^T, s_n^T) \big/ \partial s^T \geq \partial g^T(w_m^T, s_{n+1}^T) \big/ \partial s^T \tag{5.8}$$

The reason for this is that the change in storage that maintains constant utility must be larger for the contour intersecting $\left(w_m^T, s_{n+1}^T\right)$ than the contour intersecting $\left(w_m^T, s_n^T\right)$. The same applies for a fixed storage. Any contour passing through a horizontal line (fixed storage) must have a lesser slope, with respect to the origin, at $\left(w_{m+1}^T, s_n^T\right)$ than at $\left(w_m^T, s_n^T\right)$, for any $w_{m+1}^T \geq w_m^T$,

$$\partial g^T\left(w_m^T, s_n^T\right) \Big/ \partial w^T \leq \partial g^T\left(w_{m+1}^T, s_n^T\right) \Big/ \partial w^T \tag{5.9}$$

The discussion that follows refers to $g^t\left(w^{t+1}, s^{t+1}\right)$ and assumes that it has the same properties as $g^T\left(w^{T+1}, s^{T+1}\right)$ described here. Later, in Theorem 5.5, it is proved that the form of $f^t\left(w^{t+1}, s^{t+1}\right)$ is preserved over *t*, and hence so is the form of $g^t\left(w^{t+1}, s^{t+1}\right)$, as assumed here. For different functional forms and problems, modifications such as those detailed in the following sections may not be possible, so the dimensionality of the problem may mean that it can only be solved in a reasonable time using coarse discretisations of the state variables.

Now consider the process of determining the optimal release for discrete $\left(w_m^t, s_n^t\right)$ in period *t*. As discussed earlier, the basic procedure of determining the optimal release for some $\left(w_m^t, s_n^t\right)$, is to evaluate $g^t\left(w^{t+1}, s^{t+1}\right)$ for each discrete q_k^t, where $w^{t+1} = w_m^t + B^t\left(q_k^t\right)$ and $s^{t+1} = s_n^t - q_k^t$. In terms of the contours of $g^t\left(w^{t+1}, s^{t+1}\right)$, this procedure corresponds to finding the intersection of the STPC and the highest contour of $g^t\left(w^{t+1}, s^{t+1}\right)$. Figure 5.5 illustrates this intersection for the STPC associated with $\left(w_m^t, s_n^t\right)$. Also illustrated on the STPC are the release levels corresponding to the breakpoints in $B^t\left(q_k^t\right)$, where $q_1 < q_x < q_y < q_z < q_K$ are a subset of the *K* discrete release levels. In this case, the STPC intersects contour c^3 and the optimal release is q_y. Thus, $f^t\left(w_m^t, s_n^t\right) = g^t\left(w_m^t + B^t\left(q_y\right), s_n^t - q_y\right)$ and $\hat{q}^t\left(w_m^t, s_n^t\right) = q_y$. (For ease of discussion and illustration, intersections between the STPCs and contour lines will be assumed to occur at a corner point of the STPC, although, clearly, the intersection can occur at any point on the STPC).

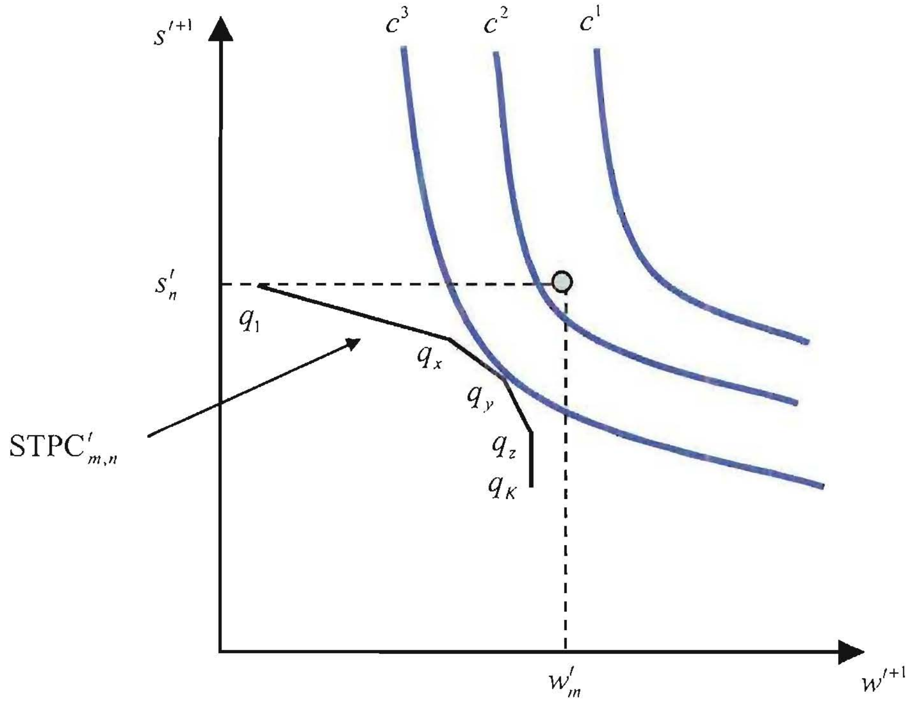


Figure 5.5: Selecting optimal release

Now consider the same procedure applied to the STPC from the same level of wealth but the next, and higher, discrete storage level i.e., point (w_m^t, s_{n+1}^t). The wealth state transition is independent of the level of storage[1], so the STPC for (w_m^t, s_{n+1}^t) has exactly the same form as used at the point (w_m^t, s_n^t) and any release feasible for (w_m^t, s_n^t) is feasible for (w_m^t, s_{n+1}^t). For storage levels less than the upper bound on release, the STPC for a higher storage level will include releases/benefits which are infeasible for lower storage levels, but this does not alter the analysis though. In terms of the placement of the STPC for (w_m^t, s_{n+1}^t) on the contour diagram, it has the same horizontal position and is shifted upwards in the storage dimension by $s_{n+1}^t - s_n^t$, as illustrated in Figure 5.6. This shows that the STPC for (w_m^t, s_{n+1}^t) intersects contour c^2 and the optimal release is q_z. Thus, $f^t(w_m^t, s_{n+1}^t) = g^t(w_m^t + B^t(q_z), s_{n+1}^t - q_z)$ and $\hat{q}^t(w_m^t, s_{n+1}^t) = q_z$.

[1] If head effects were taken into account, this would not be the case.

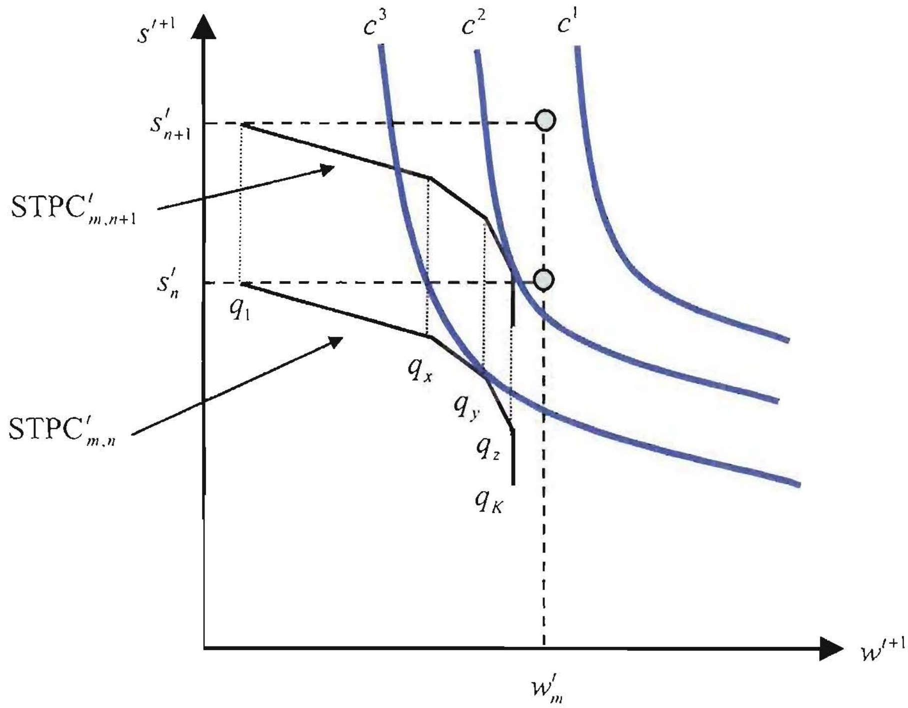


Figure 5.6: Release ratchet in storage dimension in *T*

Observe that $\hat{q}^t(w^t_m, s^t_{n+1}) \geq \hat{q}^t(w^t_m, s^t_n)$. Holding wealth constant, the slopes of the contours increase as storage increases, but the form of the STPC remains the same. In terms of searching for $\hat{q}^t(w^t_m, s^t_{n+1})$, then, it is clear that if $\hat{q}^t(w^t_m, s^t_n)$ is known, a lower bound can be defined for $\hat{q}^t(w^t_m, s^t_{n+1})$ at $\hat{q}^t(w^t_m, s^t_n)$. The search for $\hat{q}^t(w^t_m, s^t_{n+1})$ can then be restricted to $\hat{q}^t(w^t_m, s^t_n) \leq \hat{q}^t(w^t_m, s^t_{n+1}) \leq \bar{q}^t$, rather than searching over the entire range of q^t_k values i.e., $\underline{q}^t \leq \hat{q}^t(w^t_m, s^t_{n+1}) \leq \bar{q}^t$. Once $\hat{q}^t(w^t_m, s^t_{n+1})$ has been found, the same principle applies to the search for $\hat{q}^t(w^t_m, s^t_{n+2})$, and so on. In the example, given that $\hat{q}^t(w^t_m, s^t_n) = q_y$, the upper/lower bound pair are (q_y, q_K), so it is unnecessary to determine $\bar{f}^t_{m,n,k}$ for $k<y$.

Theorem 5.2. *If $f^{t+1}(w^{t+1}, s^{t+1})$ is satisfies Assumption 1 and $\hat{q}^t(w^t_m, s^t_n)$ is optimal for (w^t_m, s^t_n) then $\hat{q}^t(w^t_m, s^t_{n+1}) \geq \hat{q}^t(w^t_m, s^t_n)$ for $s^t_{n+1} > s^t_n$ and in general, $\hat{q}^t(w^t, s^t)$ will be non-decreasing for increasing s^t and w^t fixed.*

Proof. From Theorem 5.1, $g^t\left(w^{t+1}, s^{t+1}\right)$ is non-decreasing in both w^{t+1} and s^{t+1}. The STPC is constant for a change in wealth, so $B^t\left(\hat{q}^t\left(w_m^t, s_n^t\right)\right)$ will produce the same value of w^{t+1} from $\left(w_m^t, s_n^t\right)$ and $\left(w_m^t, s_{n+1}^t\right)$. Since $s_{n+1}^t > s_n^t$, $\mathrm{STPC}_{\mathrm{m,n+1}}^t$ must lie tangent to a 'higher' (or the same) contour ($\hat{c}_{m,n}$) than that which intersected $\mathrm{STPC}_{\mathrm{m,n}}^t$ at $\hat{q}^t\left(w_m^t, s_n^t\right)$. Consider now a candidate release of $q_{m,n+1}^t$ at $\left(w_m^t, s_{n+1}^t\right)$ where $q_{m,n+1}^t < \hat{q}^t\left(w_m^t, s_n^t\right)$. For $q_{m,n+1}^t$ to be optimal, the slope of contour $\hat{c}_{m,n+1}$ (the contour tangent to $\mathrm{STPC}_{\mathrm{m,n+1}}^t$) must be less than the slope of $\hat{c}_{m,n}$ at the same value of wealth. I.e., if a vertical line is drawn through the point $w_m^t + B^t\left(q_{m,n+1}^t\right)$, then $q_{m,n+1}^t$ optimal implies that the slope of the contour intersecting that vertical line at $s_{n+1}^t - q_{m,n+1}^t$ must be greater than the slope of $\hat{c}_{m,n}$ at $s_n^t - q_{m,n+1}^t$. This contradicts Equation 5.8, which identifies the slope of any contour higher than $\hat{c}_{m,n}$ as having a greater slope for a fixed value of wealth. It is therefore not possible for any point on $\mathrm{STPC}_{\mathrm{m,n+1}}^t$ to be tangent to a contour line of $g^t\left(w^{t+1}, s^{t+1}\right)$ at any point corresponding to a release less than $\hat{q}^t\left(w_m^t, s_n^t\right)$. The tangent point must occur on the STPC at some $q_{m,n+1}^t \geq \hat{q}^t\left(w_m^t, s_n^t\right)$, so $\hat{q}^t\left(w_m^t, s_{n+1}^t\right) \geq \hat{q}^t\left(w_m^t, s_n^t\right)$. The reasoning applies to any two storage levels, and hence $\hat{q}^t\left(w_m^t, s_{n+1}^t\right) \geq \hat{q}^t\left(w_m^t, s_n^t\right)$ $\forall\left(s_n^t : n, n+1 \in N\right)$. □

With $g^t\left(w^{t+1}, s^{t+1}\right)$ having the same properties with respect to wealth as it does with storage, the behaviour of the contours with wealth fixed and storage varying will be mirrored for the case where wealth can vary but storage is fixed. Consider now the case when shifting the STPC in the wealth dimension and holding storage constant i.e., a 'release ratchet' in the wealth dimension. In the same way that the slope of the contours increases relative to a vertical line (fixed wealth) and storage increasing (Figure 5.6), the slopes of the contours decreases relative to a horizontal line (fixed storage) and increasing wealth. Figure 5.7 shows the STPC for $\left(w_{m+1}^t, s_n^t\right)$, where $w_{m+1}^t \geq w_m^t$. The optimal release for $\left(w_{m+1}^t, s_n^t\right)$ occurs at q_x, which is less than the optimal release of q_y at $\left(w_m^t, s_n^t\right)$.

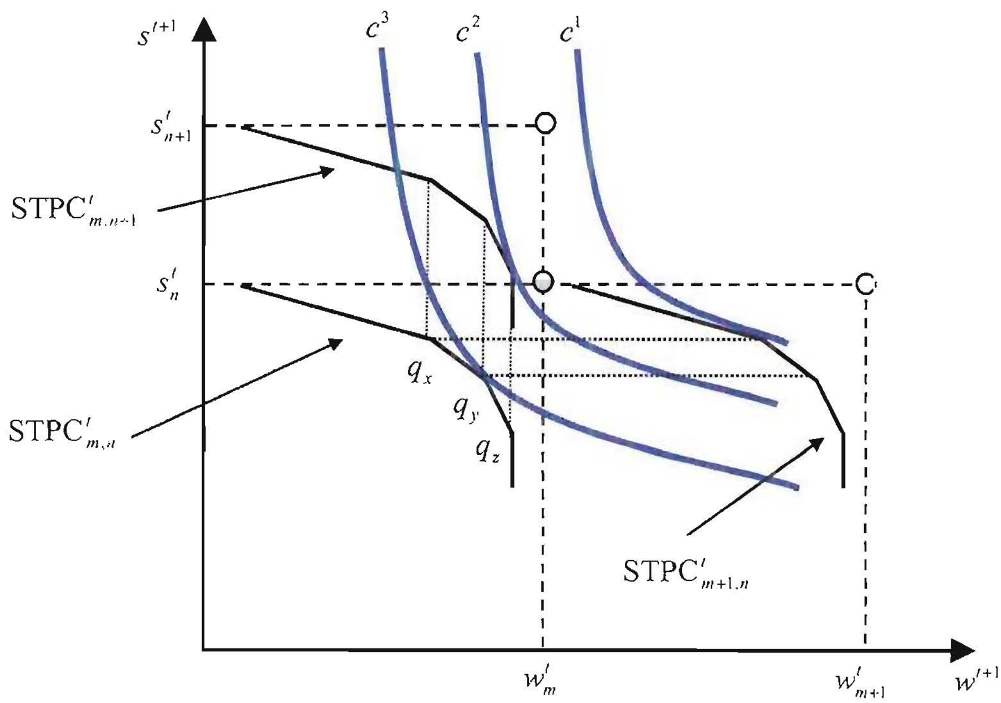


Figure 5.7: Release ratchet in storage and wealth dimensions in T

For some fixed level of storage, the marginal utility derived from accruing more wealth decreases as wealth increases, it will therefore be optimal to release less at a higher level of wealth than at a lower level of wealth because the associated increase in storage will result in a higher value of expected utility. This is the same effect which occurs when wealth is fixed and storage is varied as discussed earlier. In that case though, it was optimal to release more at the higher storage level because the increase in expected utility from higher wealth would outweigh the decrease in expected utility from lower storage.

Theorem 5.3. *If $f^{t+1}(w^{t+1}, s^{t+1})$ satisfies Assumption 1, then if $\hat{q}^t(w^t_m, s^t_n)$ is optimal for (w^t_m, s^t_n) then $\hat{q}^t(w^t_{m+1}, s^t_n) \leq \hat{q}^t(w^t_m, s^t_n)$ for $w^t_{m+1} \geq s^t_m$ and in general, $\hat{q}^t(w^t, s^t)$ will be non-increasing for increasing w^t and s^t fixed.*

Proof. From Theorem 5.1, $g^t(w^{t+1}, s^{t+1})$ is non-decreasing in both w^{t+1} and s^{t+1}. The STPC is constant for a change in storage, so $B^t(\hat{q}^t(w^t_m, s^t_n))$ will produce the same value of s^{t+1} from (w^t_m, s^t_n) and (w^t_{m+1}, s^t_n). Since $w^t_{m+1} > w^t_m$, $\mathrm{STPC}^t_{m+1,n}$ must lie tangent to a higher, or the same, contour than $\hat{c}_{m,n}$, which intersected $\mathrm{STPC}^t_{m,n}$ at $\hat{q}^t(w^t_m, s^t_n)$.

Consider now a candidate release, $q^t_{m+1,n}$, at (w^t_{m+1}, s^t_n) where $q^t_{m+1,n} > \hat{q}^t(w^t_m, s^t_n)$. For $q^t_{m+1,n}$ to be optimal, the slope of contour $\hat{c}_{m,n+1}$ (the contour tangent to $\mathrm{STPC}^t_{m+1,n}$) must be greater than the slope of $\hat{c}_{m,n}$ at the same value of storage. I.e., if a horizontal line is drawn through the point $s^t_n - q^t_{m+1,n}$, then $q^t_{m+1,n}$ optimal implies that the slope of the contour on this line at $w^t_{m+1} + B^t(q^t_{m+1,n})$ must be greater than the slope of $\hat{c}_{m,n}$ at $w^t_m + B^t(q^t_{m+1,n})$. This contradicts Equation 5.9, which identifies the slope of any contour higher than $\hat{c}_{m,n}$ as having a lesser slope for a fixed value of storage. It is therefore not possible for any point on $\mathrm{STPC}^t_{m+1,n}$ to be tangent to a contour line of $g^t(w^{t+1}, s^{t+1})$ at any point corresponding to a release greater than $\hat{q}^t(w^t_m, s^t_n)$. The tangent point must occur on the STPC at some $q^t_{m+1,n} \le \hat{q}^t(w^t_m, s^t_n)$, so $\hat{q}^t(w^t_{m+1}, s^t_n) \ge \hat{q}^t(w^t_m, s^t_n)$. The reasoning applies to any two wealth levels, and hence $\hat{q}^t(w^t_{m+1}, s^t_n) \ge \hat{q}^t(w^t_m, s^t_n)\ \forall (w^t_m : m, m+1 \in M)$. □

From these results it follows that the release ratchet can be applied simultaneously in the wealth and storage dimensions, and hence to define upper and lower bounds on $\hat{q}^t(w^t_{m+1}, s^t_{n+1})$.

Theorem 5.4. *If $f^{t+1}(w^{t+1}, s^{t+1})$ satisfies Assumption 1 and given that $\hat{q}^t(w^t_{m+1}, s^t_n)$ is optimal for (w^t_{m+1}, s^t_n) and $\hat{q}^t(w^t_m, s^t_{n+1})$ is optimal for (w^t_m, s^t_{n+1}), the bounds on $\hat{q}^t(w^t_{m+1}, s^t_{n+1})$ are $\hat{q}^t(w^t_{m+1}, s^t_n) \le \hat{q}^t(w^t_{m+1}, s^t_{n+1}) \le \hat{q}^t(w^t_m, s^t_{n+1})$. In general, $\hat{q}^t(w^t, s^t)$ will be non-decreasing for s^T increasing and w^T decreasing.*

Proof. This follows directly from Theorem 5.2, which proves that $\hat{q}^t(w^t_{m+1}, s^t_{n+1}) \le \hat{q}^t(w^t_m, s^t_{n+1})$ and from Theorem 5.3, which proves that $\hat{q}^t(w^t_{m+1}, s^t_{n+1}) \ge \hat{q}^t(w^t_{m+1}, s^t_n)$. As to the properties of $\hat{q}^t(w^t, s^t)$, Theorem 5.2 proved that $\hat{q}^t(w^t, s^t)$ is non-decreasing for w^t fixed and s^t increasing. Theorem 5.3 proved that $\hat{q}^t(w^t, s^t)$ is non-increasing for s^t fixed and w^t increasing, so $\hat{q}^t(w^t, s^t)$ is non-decreasing for s^t fixed and w^t decreasing. Thus $\hat{q}^t(w^t, s^t)$ is non-decreasing for s^t increasing and w^t decreasing. □

All that remains to be shown is that $\hat{q}^t(w^t, s^t)$ will preserve the properties of $f^t(w^t, s^t)$ between the periods. If the STPC originates from a higher storage (wealth) level the STPC will intersect a higher contour of $g^t(w^{t+1}, s^{t+1})$; an equal or higher expected utility must be able to be achieved if more storage (wealth) is available and the level of wealth (storage) is the same. In fact, the form of the optimal release function will preserve the properties of $f^t(w_m^t, s_n^t)$ required for the solution to the SDP to be optimal.

Theorem 5.5. *If $f^{t+1}(w^{t+1}, s^{t+1})$ is defined according to Assumption 1 and $\hat{q}^t(w^t, s^t)$ is non-decreasing for s^t increasing and w^t decreasing, $f^t(w^t, s^t)$ is non-decreasing at a non-increasing rate and thus preserves the properties of $g^t(w^{t+1}, s^{t+1})$.*

Proof. See Hadley (1964, Ch. 10). □

The ability to 'lock in' a release level as the state space is searched (via the 'release ratchet' approach) means that the search for the optimal release for some (w_m^t, s_n^t) can be started from the 'ratcheted' release level rather than from $\underline{q}^t$. For a given storage and wealth, this release level is defined as the maximum of the optimal release at the lower storage level (same wealth) and the release level at the higher wealth level (same storage). The release ratchet has been incorporated into the solution algorithm shown in Figure 5.8.

Determine $f^{T+1}\left(w_m^T, s_n^T\right)$
Create $g^t\left(w^{t+1}, s^{t+1}\right)$ (see Figure 5.3)
$rr_n = 0 \; \forall n$
For $t = T \ldots 1$ (period)
    For $m = M \ldots 1$ (wealth w_m^t)
        For $n = 1 \ldots N$ (storage s_n^t)
            $f^t\left(w_m^t, s_n^t\right) = -\infty$
            $\bar{k} = \max\left(rr_n, rr_{n-1}\right)$
            For $k = \bar{k} \ldots K$ given $q_k^t < s_n^t$ (release q_k^t)
                $w^{t+1} = w_m^t + B^t\left(q_k^t\right)$
                $s^{t+1} = s_n^t - q_k^t$
                $f_{m,n,k}^t = g^t\left(w^{t+1}, s^{t+1}\right)$
                If $\bar{f}_{m,n,k}^t > f^t\left(w_m^t, s_n^t\right)$
                    $f^t\left(w_m^t, s_n^t\right) = \bar{f}_{m,n,k}^t$
                    $\hat{q}^t\left(w_m^t, s_n^t\right) = q_k^t$
                    $rr_n = k$
                end if
            end for k
        end for n
    end for m
end for t

Figure 5.8: Solution algorithm with release ratchets

The variable $rr_n \;\; n \in N$ stores the value of k corresponding to the optimal release for storage level s_n^t. Because rr_n is progressively updated, when a new iteration of the loop begins, the value of rr_n will correspond to the optimal release for storage s_n^t and wealth w_{m-1}^t, while as the update progresses, rr_{n-1} corresponds to the optimal release for storage s_{n-1}^t and wealth level w_m^t, hence the ratchet release index for the storage and wealth pair is defined as $\bar{k} = \max\left(rr_n, rr_{n-1}\right)$. Note also that the wealth variable is searched in order of 'best' to 'worst', so $m = M \ldots 1$ rather than $m = 1 \ldots M$ as in the previous algorithms. This allows the max operator to be used on the optimal release indices to define a lower bound on the current optimal release, with the upper bound remaining at $\bar{q}^t$. The reason for this is that the 'search termination' scheme discussed in the next section handles that aspect of the search, so any reductions in the range of q_k^t required to be searched would not have been utilised.

If the algorithm were searching in the storage dimension while holding wealth fixed, and the 'release ratchet' was only applied in the storage dimension, then the search for $\hat{q}^t\left(w_m^t, s_{n+1}^t\right)$ would start from $\hat{q}^t\left(w_m^t, s_n^t\right)$ rather than $\underline{q}^t$, so $\bar{f}_{m,n,k}^t$ would be calculated about $M \times N \times \frac{K}{2}$ times per period, rather than $M \times N \times K$ times, indicating approximately a 50% reduction in the number of interpolations required to determine $\hat{q}^t\left(w^t, s^t\right)$ and $f^t\left(w^t, s^t\right)$, assuming that the distribution of optimal releases is evenly distributed over S^t. If the 'wealth ratchet' were applied separately, a similar saving would be expected. When the release ratchet is applied in both the storage and wealth dimensions, then there will presumably be some small additional saving from starting the search for $\hat{q}^t\left(w_{m+1}^t, s_{n+1}^t\right)$ at the maximum of the optimal releases for the adjacent storage and release grid points, though it is not obvious a priori what the magnitude of this saving will be.

Table 5.1 shows the solution times of a moderately sized problem with *M*=200, *N*=200, *K*=500, and *T*=52. The problems were run on a Pentium233 (these times could no doubt be improved were the code to be streamlined for execution speed).

	No ratchet	Ratchet in W	Ratchet in S	Ratchet in W&S
Time (mins)	77.02	32.16	32.30	30.16
Improvement		58%	58%	61%

Table 5.1: Effect of 'release ratchet' on execution time

Using the ratchet reduced the solution time by about 60% from the 'no ratchet' solution time. Note too that the times quoted here all include the improved handling of the inflow adjustment. Were this excluded, the times would be approximately 5 times longer.

5.4.3 Search termination

The computational effort required to determine $\hat{q}^t\left(w^t, s^t\right)$ can be reduced even further by utilising the form of $B^t\left(q^t\right)$, which is non-decreasing at a non-increasing rate (first derivative). Therefore, it is possible to terminate the search for $\hat{q}^t(w^t, s^t)$ because it equates to finding the unique intersection of $B^t(q^t)$ (concave) and the highest contour (convex) of $g^t(w^{t+1}, s^{t+1})$. The resulting values of $\bar{f}^t_{m,n,k}$ are therefore concave in release q^t_k and, because $\hat{q}^t(w^t_m, s^t_n)$ is found by evaluating $\bar{f}^t_{m,n,k}$ for discrete q^t_k, the search can be terminated once $\bar{f}^t_{m,n,k}$ starts to decrease. Note that, from Theorem 5.4, the starting point of the search.

Theorem 5.6. *If $f^{t+1}(w^{t+1}, s^{t+1})$ is satisfies Assumption 1, and $\hat{q}^t(w^t_m, s^t_n)$ is optimal for (w^t_m, s^t_n), then for some (w^t_m, s^t_{n+1}), the search for $\hat{q}^t(w^t_m, s^t_{n+1}) \geq \hat{q}^t(w^t_m, s^t_n)$ can be terminated once $\bar{f}^t_{m,n+1,k+1} \leq \bar{f}^t_{m,n+1,k}$.*

Proof. From Theorem 5.1, $g^t(w^{t+1}, s^{t+1})$ is non-decreasing in both w^{t+1} and s^{t+1}, so the contours of $g^t(w^{t+1}, s^{t+1})$ are therefore convex to the origin. The STPC is concave with respect to the origin, so the value of $g^t(w^{t+1}, s^{t+1})$ corresponding to each point on the STPC is therefore concave with a single optimal solution. This equates to maximisation of a concave function as discussed in Hadley (1964 Ch. 10-11), for example. If the search for $\hat{q}^t(w^{t+1}_m, s^{t+1}_n)$ is in one direction over the range of q^t_k, and this range includes $\hat{q}^t(w^{t+1}_m, s^{t+1}_n)$, the search can be stopped once the value of $g^t(w^{t+1}, s^{t+1})$ associated with q^t_k is less than value of $g^t(w^{t+1}, s^{t+1})$ associated with q^t_{k-1}. It follows then that $\hat{q}^t(w^{t+1}_m, s^{t+1}_n) = q^t_k$. □

Figure 5.9 shows how the algorithm is changed to accommodate 'early termination'. The existing code is on the left with the new code on the right. There are several ways the 'search termination' could be implemented. Using the release ratchet modification, the *k* loop starts from the maximum of the optimal release at the lower storage level, or at the same storage level but higher wealth. This is a lower bound on the optimal release level as proved earlier. The modification simply involves setting the release index loop to *K+1* once $\bar{f}^t_{m,n,k} < f^t(w^t_m, s^t_n)$, which has the effect of ending the k loop when k is next evaluated against the upper bound in the *for* loop (see Figure 5.8).

If $\bar{f}^t_{m,n,k} > f^t(w^t_m, s^t_n)$
$f^t(w^t_m, s^t_n) = \bar{f}^t_{m,n,k}$
$\hat{q}^t(w^t_m, s^t_n) = q^t_k$
$rr_n = k$
end if

If $\bar{f}^t_{m,n,k} > f^t(w^t_m, s^t_n)$
$f^t(w^t_m, s^t_n) = \bar{f}^t_{m,n,k}$
$\hat{q}^t(w^t_m, s^t_n) = q^t_k$
$rr_n = k$
else
$k = K + 1$
end else

Figure 5.9: Search termination modifications

The effect of the search termination is difficult to quantify, as it depends on the form of $B^t(q^t_k)$ and its interaction with $g^t(w^{t+1}, s^{t+1})$. Table 5.2 shows the execution times that resulted with and without the search termination (ET) modification and using the same model as used for the release ratchet tests.

	No ratchet	Ratchet in W	Ratchet in S	Ratchet in W&S
Without ST	77.02	32.16	32.30	30.16
With ST	45.43	0.88	0.95	0.63
Improvement from using ST	*41%*	*98%*	*98%*	*99%*
Overall improvement		***99%***	***99%***	***99%***

Table 5.2: Effect of 'early termination' rule on execution time

When the 'search termination' and 'release ratchet' modifications are implemented in tandem, the reduction in CPU time required for the optimisation is approximately 99% of that required without the modifications.

5.5 Conclusions

This chapter has dealt with implementation issues relating to SUMDP, in particular, for the case of 'regulated' reservoir management. Because SUMDP augments the state space with the accumulated wealth variable, the computational requirements are demanding since the potential number of discrete points evaluated is the square of the number of discrete points in each dimension. It has been shown that the form of the benefit function and utility function may mean that the basic solution algorithm, which can quickly become computationally intractable, can be modified to take advantage of their properties and yield improvements in computation time of up to 99% from the requirements of the unmodified algorithm. For reservoir management in a regulated environment, it is not an unreasonable assumption that the benefit and utility functions would have the form required to perform these modifications i.e., that cost decreases as release increases, and that a non-decreasing utility function with non-increasing first derivatives reflects a DM's trade-off between increasing storage and decreasing accumulated cost. In the next chapter, results are discussed for such a reservoir management problem where a DM is trading-off (via a utility function) storage and accumulated 'non-hydro' cost in order to meet demand.

Chapter 6

Experimental Results for Regulated Reservoir Management

6.1 Introduction

In this chapter, optimisation and simulation results from a medium-term (one year) reservoir scheduling problem are explored using a representation of the New Zealand electricity system. The scenario considered here is described as a regulated electricity system, as described in the previous chapter, and a situation which existed in New Zealand up to 1992. The benefit function in each period takes the form described in the previous chapter, so the benefit, or return, from release reflects the cost of satisfying the residual demand from available generation sources; the residual demand is that which remains after hydro release has been subtracted. Concave and non-decreasing bi-variate utility functions with different curvatures are defined over the range of end-of-horizon accumulated costs and storage levels. The optimisation and simulation results are then compared to each other and to a base case where the terminal value function is defined as the value of storage, which is the form of value function used in most standard SDP approaches to this problem. The layout of this chapter is as follows:

- The (risk neutral) base case and utility functions are discussed in Section 6.1.
- End-of-horizon performance is discussed in Section 6.2.
- Weekly performance is discussed in Section 6.3.

6.2 System data and the base case

The model is tested using a representation of the New Zealand electricity system. Most of the data used to derive these results is taken from Scott (1997). The supply curve is derived from thermal marginal costs and generation capacities, and run-of-river (uncontrollable) hydro. The capacities and marginal costs of these stations are detailed in Table 6.1.

Station	Capacity (MWh)	Marginal Cost ($/MWh)
Manapouri	570	5
Clutha	639/244	5
Huntly	900	20
New Plymouth	518	25
Stratford	178	35
Marsden A	103	60
Otahuhu	81	110
Whirinaki	194	150

Table 6.1: Supply curve data

Manapouri and Clutha are hydro stations which are treated as run-of river, so their marginal cost is nominal. Clutha's weekly capacity is determined by interpolating between a summer capacity of 639MWh and a winter capacity of 244MWh. Because demand can not be met entirely from thermal plant, a 'shortage' (or VoLL/value of lost load) station is also included, which has a constant marginal cost of $500/MWh and an unlimited capacity. An example of the residual demand curve (RDC) and benefit function is shown in Figure 6.1, where the demand and supply data corresponds to period 2, with d^t=3230MWh. As described in the previous chapter, the RDC in a given period was created by subtracting the supply curve from the fixed demand curve.

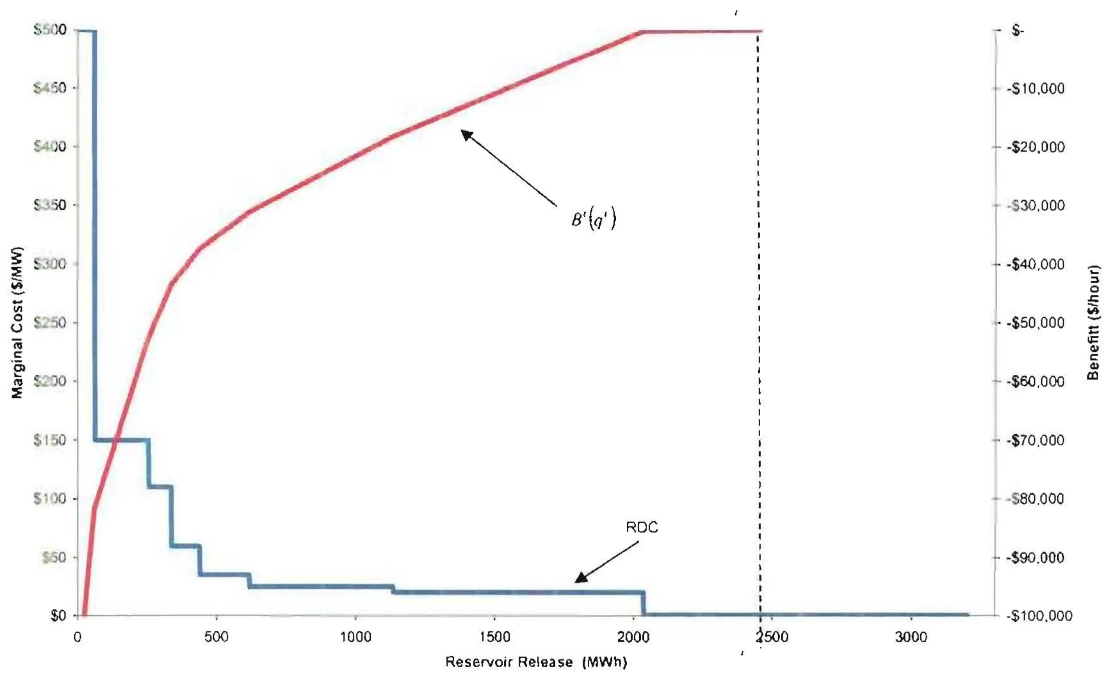


Figure 6.1: RDC and benefit curve

For ease of comparison with the results in , a single demand curve was used to approximate the weekly load duration curve. The weekly demand corresponded to the middle (third) sub-period demand in Scott (1997), who approximated the weekly load duration curve with 5 equally weighted sub-periods. A fixed demand curve is used here whereas Scott used constant elasticity demand curves. Demand in each period was calculated by interpolating between summer (week 1) and winter (week 26) demand levels, which were 3204MWh and 3864MWh, respectively, somewhat lower than those used by Scott. However, the distributions of EOH-W and EOH-S using the single fixed demand curve were very similar to those obtained by Scott using a five sub-period approximation with (fixed demand in each sub-period). The reason for this is that, not surprisingly, the aggregated RDCs (created by summing the weighted sub-period RDCs) were similar to the RDCs created using the demand of the middle sub-period. Because the aggregated RDCs generally lay to the left of the single demand RDCs, lower costs were incurred for the same release, and the EOH-W distributions shifted to the right.

Figure 6.2 shows the inflow data in the form of a mean and standard deviation for each week (in GWh).

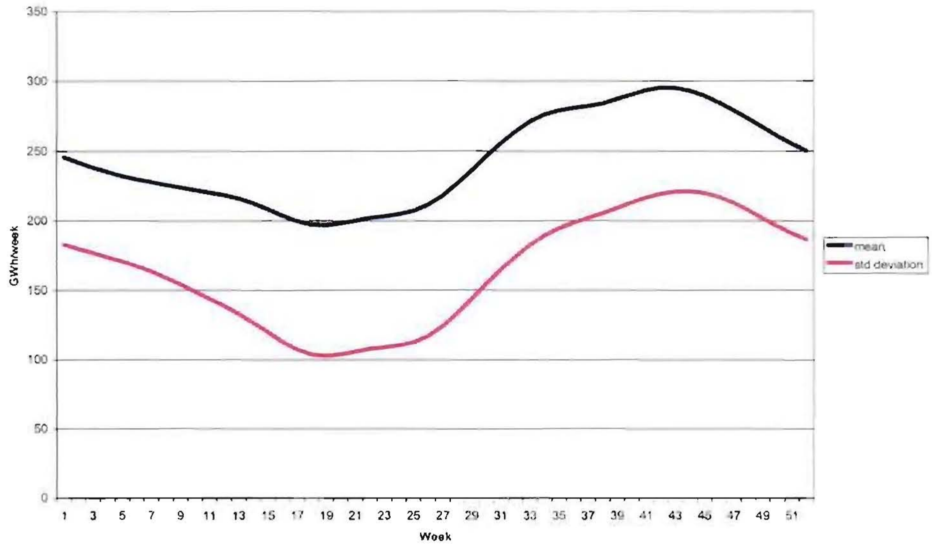


Figure 6.2: Inflow data

For simplicity, it is assumed that a unit of inflow corresponds to a unit of storage i.e., that the inflow/storage efficiency ratio = 1. Inflows are assumed to be Normally distributed with a known mean and variance, and approximated by a discrete distribution with 5 points with probabilities of 0.15, 0.2, 0.3, 0.2, and 0.15, respectively. Maximum storage was 2900 GWh and minimum storage was 0 GWh. Maximum generation/release was 2455MWh and minimum generation/release was 0MWh. It was also assumed that a unit of release corresponds to a unit of electricity supplied to the market i.e., that the release/generation efficiency ratio = 1.

The optimisation and simulation code was written in C and compiled/executed on a 266MHZ/128MB PC. The grid discretisations were 500 for storage (corresponding to 5.8GWh storage steps), 1000 for wealth, and 2456 for release (corresponding to 1MWh release steps). The following bounds were used on wealth for all optimisations: $\overline{W}_H^1$ = -\$1m, $\overline{W}_H^T$ =-\$50m, $\underline{W}_H^1$=-\$150m, $\underline{W}_H^T$=-\$250m (refer to Chapter 5 for the definition and discussion of these parameters). The grid spacing in the storage dimension was constant throughout the optimisation. Approximately 4 minutes was required for the optimisation, while the simulation time (20 simulations) was negligible. Initial storage for each simulation was 1450GWh. The random number generator used is that described in Law and Kelton (1991); Scott used *Matlab*'s random number generator. A simple test of 1-dimensional uniformity was performed for a variety of sample sizes. The test statistics were less than the 5% chi-square values for all sample sizes.

The base case for these experiments is where the player is risk neutral (RN) with respect to wealth. This corresponds to the perfectly competitive (PC) solution in Scott (1997). The end-of-horizon (non-increasing) marginal water value curve was taken from his model and the corresponding (non-decreasing) water value curve used as the end-of-horizon value function for the SDP. The graphs used to depict results for a single variable have five lines, where the points on these lines correspond to the 5th, 25th, 50th, 75th, and 95th percentile values taken from the simulation results (20 simulations). Figure 6.3 shows the optimal weekly release curves, which are, for this RN case, a function of the storage level alone and determined in the optimisation. From the simulation results, the weekly release, weekly storage, and mean weekly generation levels of other firms, are shown in Figure 6.5, Figure 6.4, and Figure 6.6, respectively.

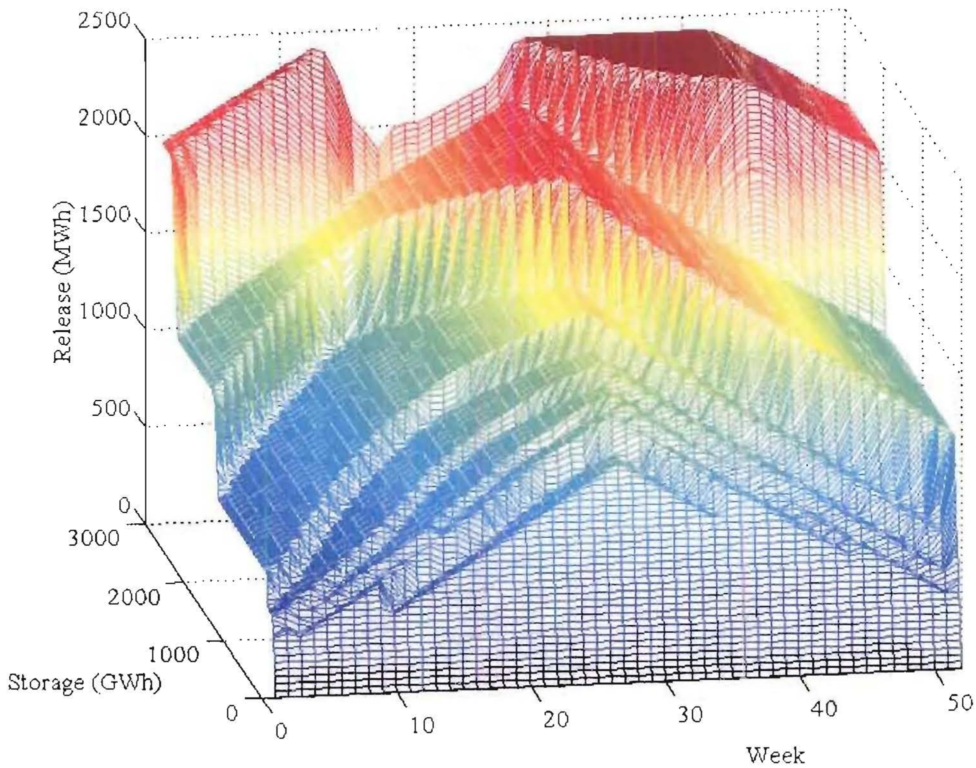


Figure 6.3: RN optimal release surface

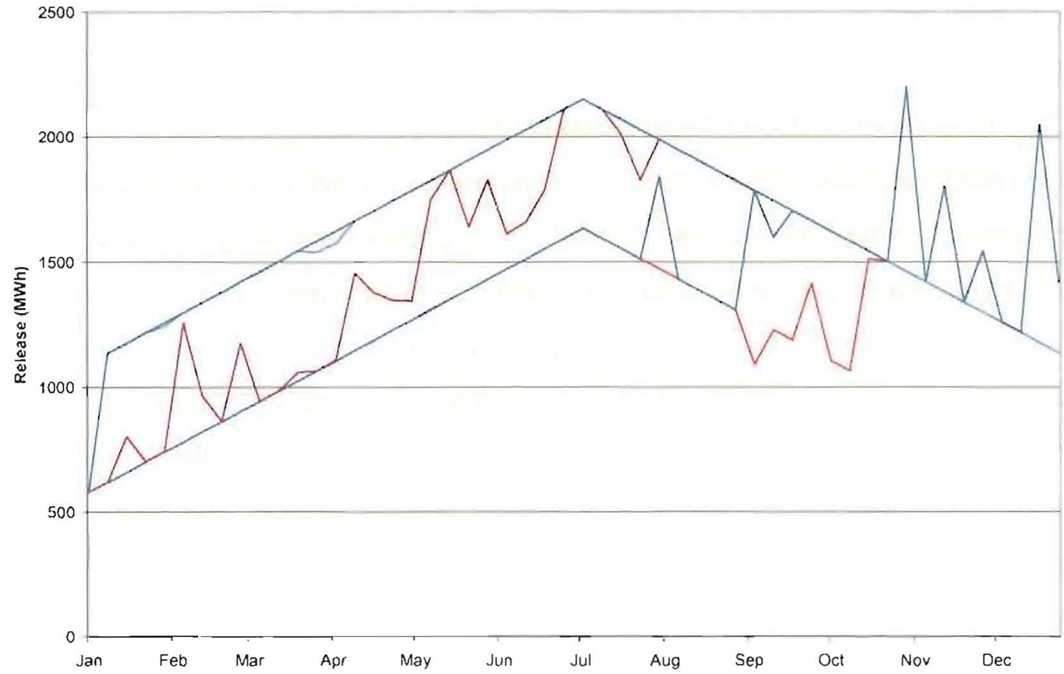


Figure 6.4: RN reservoir release

Figure 6.5: RN storage

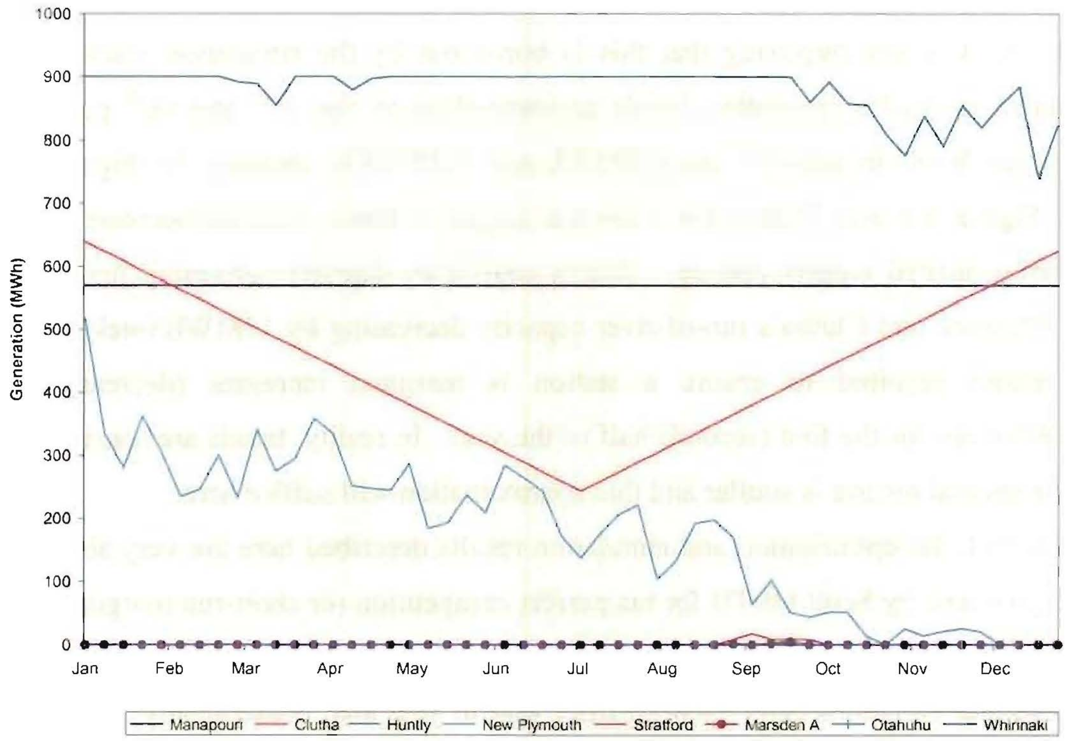


Figure 6.6: RN other firm generation

All simulations start from the same initial storage level of 1450GWh, and the values of storage (and wealth) are those at the end of the period, after the release decision has been made and an inflow observed. This is the reason that there is variability in the storage level in the first week of the simulation (Figure 6.5), but no variability in the associated release decision (Figure 6.4). Release generally follows the demand trajectory. A flat segment on the optimal release curve produces a plateau or 'terrace' on the optimal release surface, corresponding to a single release level being optimal for a range of storage levels. These release levels correspond to corner points on the benefit curve, and hence to all thermal stations up to a certain marginal cost generating at their minimum or maximum with the remainder at their minimum. It is worth noting that the drop in optimal release levels for high storage during weeks 20-30 is due to the interaction of inflows (decreasing) and demand (increasing). For that set of periods, it is economic to withhold release and incur higher thermal costs so as to avoid even higher thermal costs in later periods.

Referring to the optimal release surface, $q^2 = 618$ for $279 \leq s^2 \leq 1575$, and $q^2 = 1135$ for $1674 \leq s^2 \leq 2557$, with these generation levels being the minimum and maximum releases that will ensure New Plymouth is the marginal station with marginal cost \$25/MWh. Also, these two release levels are optimal for a wide range of storage levels, so it is not surprising that this is borne out by the simulation results. For example, the hydro generation levels corresponding to the 10^{th} and 90^{th} percentile generation levels in period 2 are 618MWh and 1135MWh, respectively (Figure 6.4). Both Figure 6.3 and Figure 6.4 illustrate ranges a linear increase/decrease in the optimal/simulated weekly release. This is caused by demand increasing linearly by 25MWh/week and Clutha's run-of-river capacity decreasing by 15MWh/week, so that the release required to ensure a station is marginal increases (decreases) by 40MWh/week for the first (second) half of the year. In reality, trends are less definite, but the general picture is similar and this approximation will suffice here.

Overall, the optimisation and simulation results described here are very similar to those produced by Scott (1997) for his perfect competition (or short-run marginal cost) case, although Scott employed Dual DP and his simulation consists of 20 consecutive years so that variations carry forward from year to year and, consequently, his release and storage trajectories exhibited more variability. Given that that the water value, inflow, demand, and supply curve data are the only areas of commonality between the two models, this is an encouraging result in terms of the calibration of these primal SDP and dual SDP models.

6.3 Risk averse cases

We now consider cases with utility curves defined over the range of RN EOH-W and EOH-S simulated outcomes. Figure 6.7 shows the CDF of the RN EOH-W values from the simulation. The EOH-S CDF is shown in Figure 6.8. EOH-W values fall in the range -\$230m to -\$170m, so the utility curves were defined such that the significant changes in marginal utility occurred over this range. Were they not, the effect of the utility function would be minimal because there would be little change in marginal utility over the relevant range of EOH-W values. Storage ranges from about 500MW to 2500MW, so the utility functions were defined over the entire range of storage levels. The y-axis (utility) was arbitrarily defined over the range 0 to 1,000,000. Five utility curves were defined over the ranges of EOH-W and EOH-S values (Figure 6.9) and labelled from 0 to 4. For convenience, the same curves will be used in each dimension, but scaled accordingly. Curve 0 has the least curvature (it is linear) while curve 4 has the most curvature.

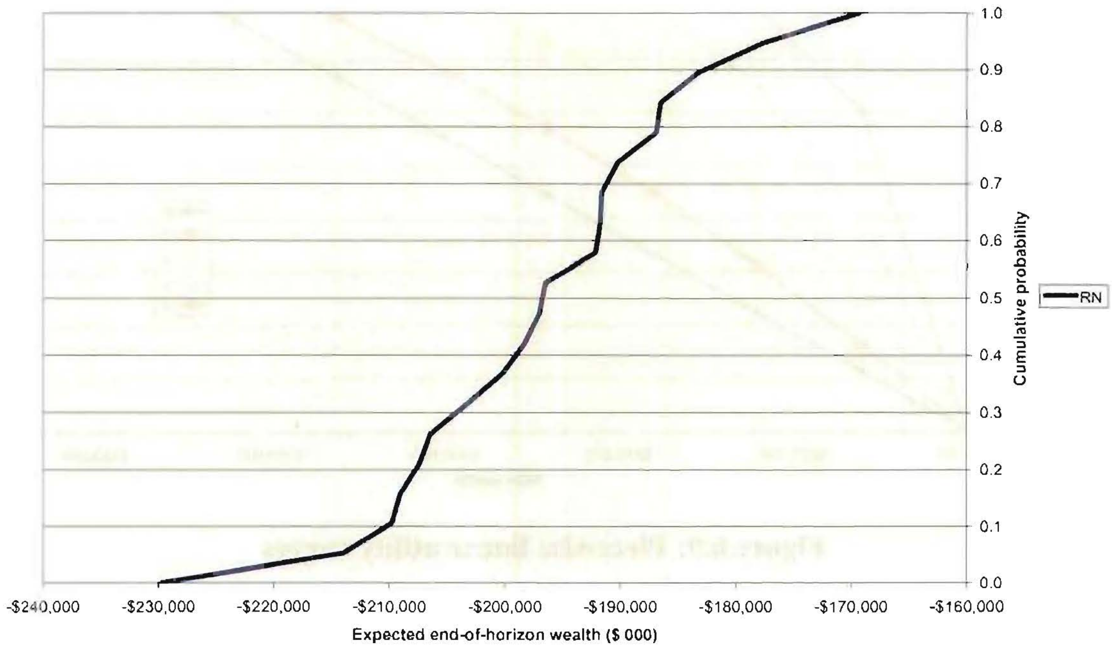


Figure 6.7: CDF of RN EOH wealth values

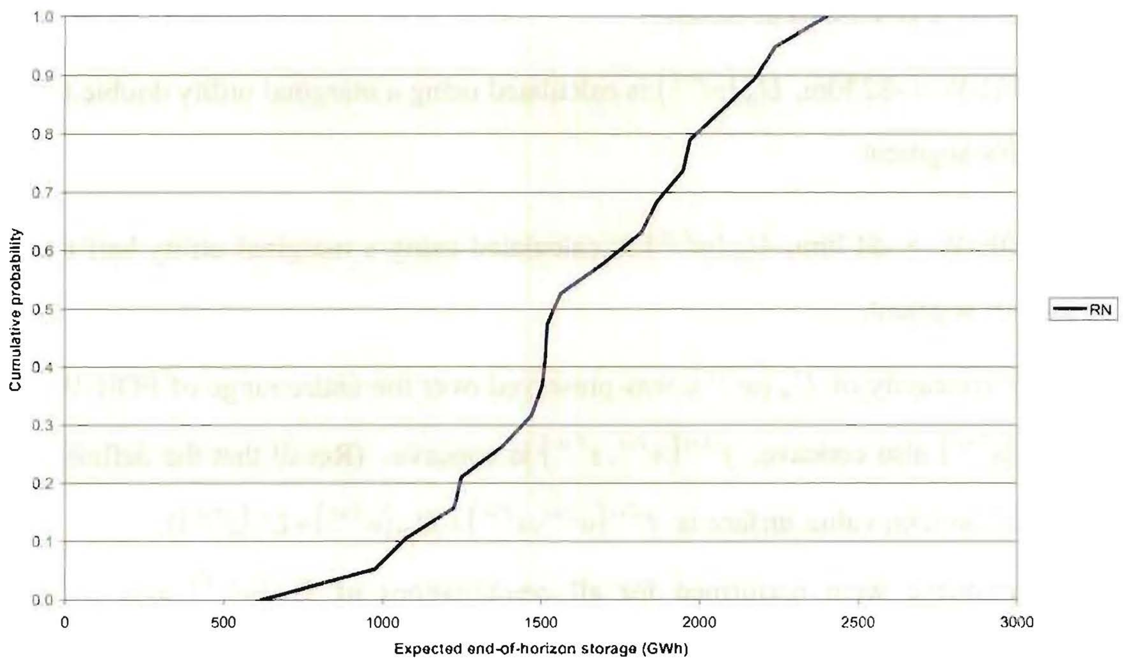


Figure 6.8: CDF of RN EOH storage values

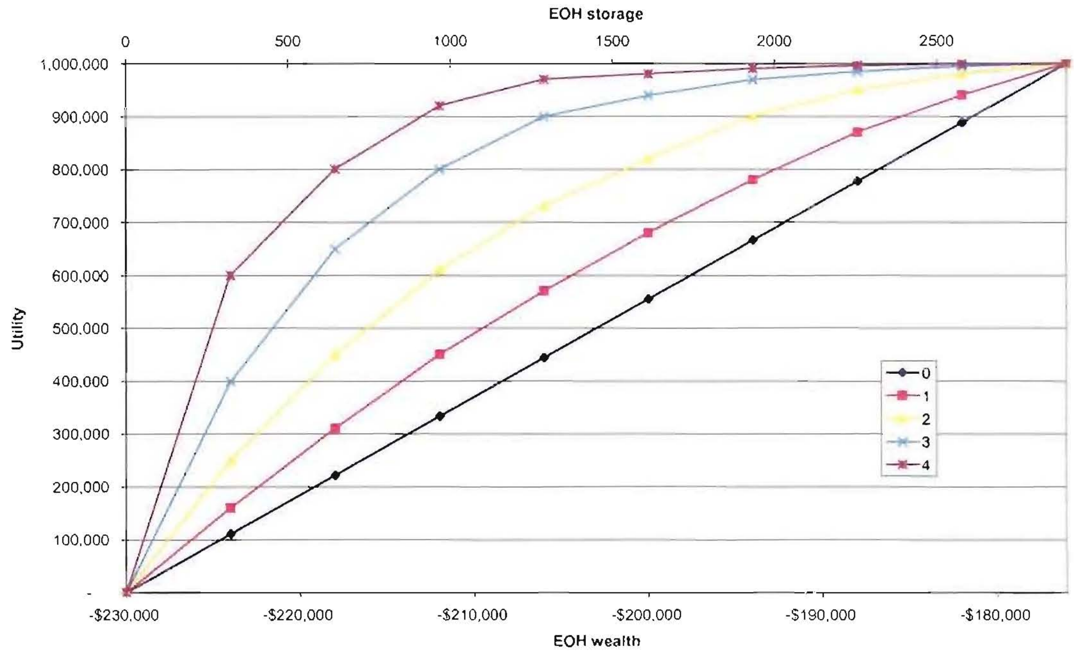


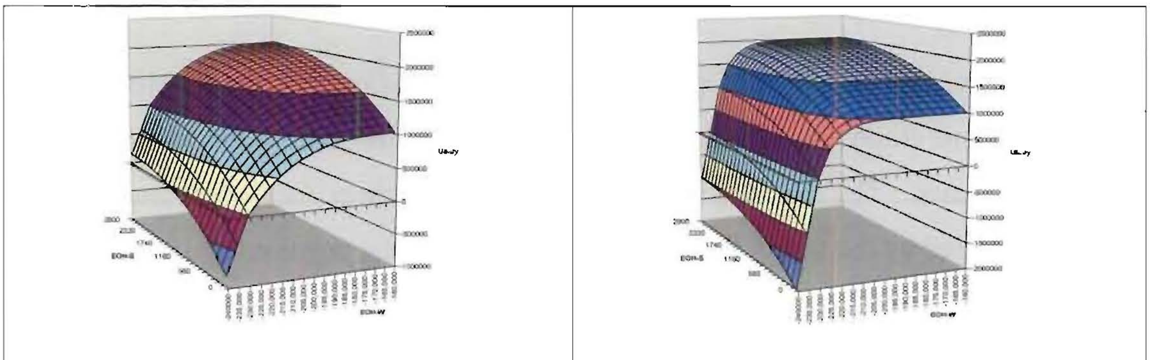
Figure 6.9: Piecewise linear utility curves

The end-of-horizon value surface, $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$, is defined over the entire range of EOH-W and EOH-S, yet the utility functions in Figure 6.9 are only defined over a smaller range of EOH-W and EOH-S. Values of U_W corresponding to EOH-W outside this range were calculated as follows:

- If EOH-W < -\$230m, $U_W\left(w^{T+1}\right)$ is calculated using a marginal utility double that of the first segment.
- If EOH-W > -\$176m, $U_W\left(w^{T+1}\right)$ is calculated using a marginal utility half that of the last segment.

Thus, the concavity of $U_W\left(w^{T+1}\right)$, was preserved over the entire range of EOH-W, and with $U_S\left(s^{T+1}\right)$ also concave, $f^{T+1}\left(w^{T+1}, s^{T+1}\right)$ is concave. (Recall that the definition of the end-of-horizon value surface is $f^{T+1}\left(w^{T+1}, s^{T+1}\right) = U_W\left(w^{T+1}\right) + U_S\left(s^{T+1}\right)$).

Experiments were performed for all combinations of $U_W\left(w^{T+1}\right)$ and $U_S\left(s^{T+1}\right)$ described above. The results obtained using the curves illustrated in Figure 6.10 receive the most attention as they reflect a 'moderate' and 'extreme' relative risk aversion towards wealth.



W2S2 | W4S2

Figure 6.10: Bi-variate utility curves

The notation used to refer to the utility functions is WxSy where x and y are the utility curves defined in Figure 6.9 for wealth and storage, respectively. Thus W4S2 means that utility curve 4 is used for wealth and utility curve 2 is used for storage. The EOH value surface is then calculated by summing the utility associated with each discrete combination of wealth and storage.

6.4 End-of-horizon performance

In this section, the impact of the bi-variate utility functions on EOH-W and EOH-S is discussed. The purpose of this discussion is two-fold; firstly to examine the effect of different strengths of relative risk aversion, and secondly, to examine the relationships between the EOH results, regardless of which utility function was used to derive them. Cumulative distribution functions (CDFs) of EOH-W and EOH-S are shown in Figure 6.11 and Figure 6.12, respectively. Summary statistics for EOH-W and EOH-S are shown in Table 6.2. Each figure contains three curves, corresponding to the RN, W2S2, and W4S2 cases. These effects are evident in the CDF plots, where the W4S2 EOH-W CDF lies to the left of the corresponding W2S2 CDF. Because W4S2 has a stronger relative wealth risk aversion (relative to storage) than W2S2, W4S2 EOH-W would be expected to be less variable (and hence more vertical) than that derived using W2S2, and this would have the associated effect of a reduced the mean in order to achieve the lower variance. Compared to the RN case, W4S2 and W2S2 both have a stronger relative wealth risk aversion, so RN EOH should be more variable. These expectations are reversed with respect to storage. W2S2 has a stronger relative storage risk aversion than W4S2, so W2S2 EOH-S would be expected to be less variable than the W4S2 values.

Stronger relative wealth risk aversion has the effect of making the EOH-W CDFs less variable and closer to vertical, and in turn, reducing the variability, and range, of EOH-W outcomes. The differences in the relative preferences for wealth and storage outcomes implied by the utility functions are reflected by the positioning and shape the CDFs.

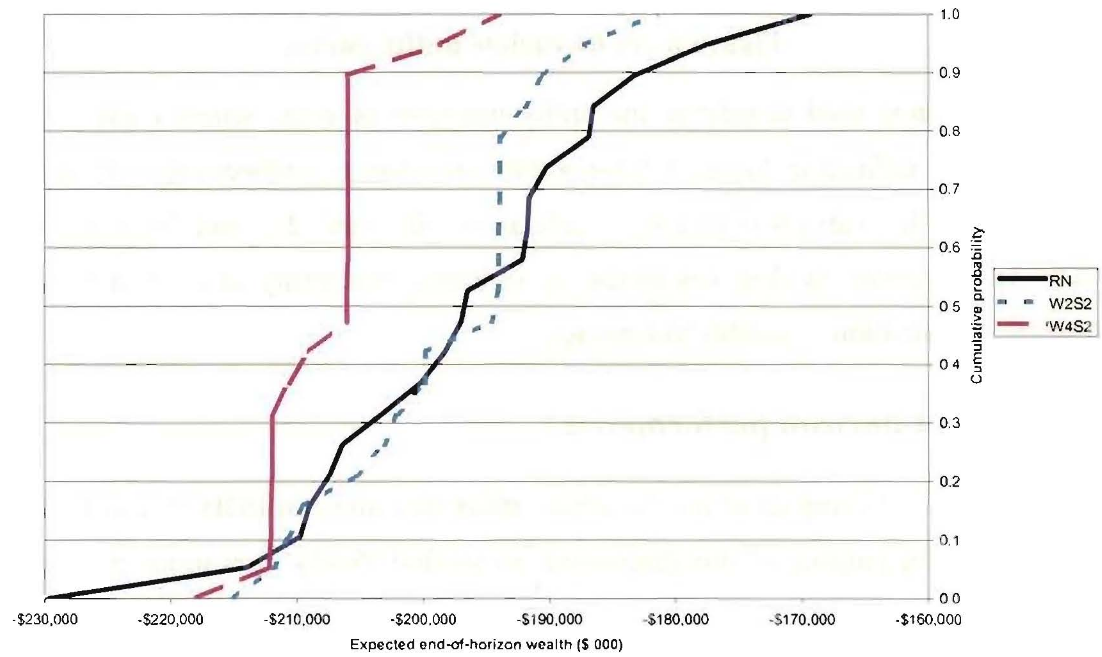


Figure 6.11: EOH wealth CDFs

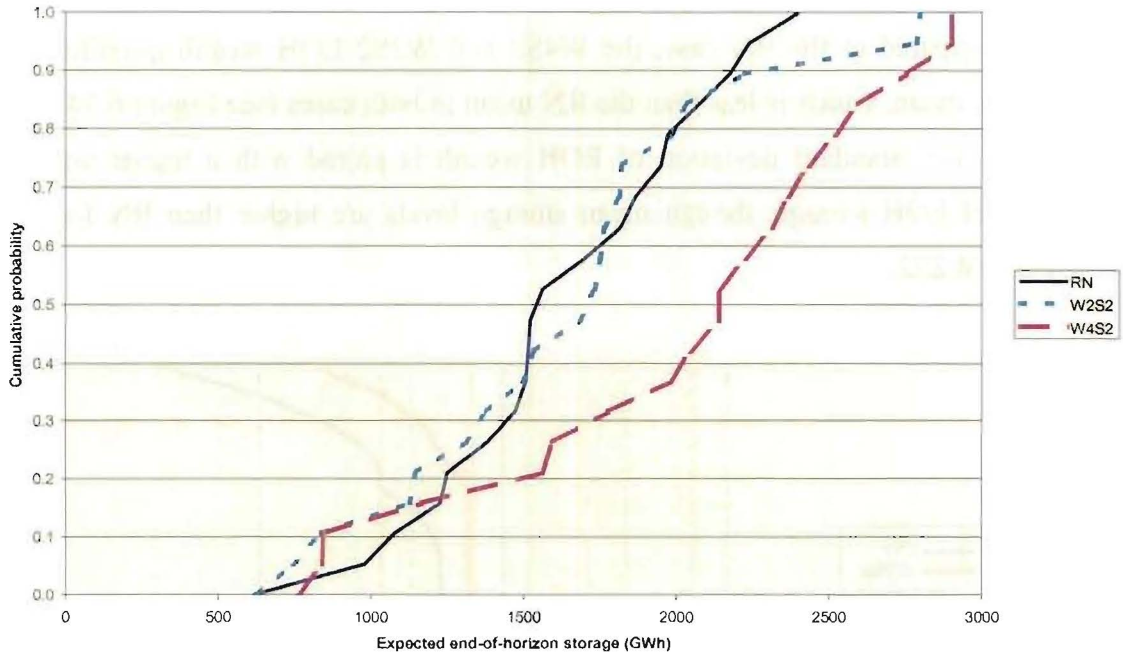


Figure 6.12: EOH storage CDFs

	RN	W2S2	W4S2
W: Mean	(197,052)	(198,355)	(207,829)
W: Standard deviation	13,733	8,781	5,270
W: Semi-deviation	15,429	10,173	5,250
S: Mean	1,614	1,629	1,991
S: Standard deviation	457	587	674

Table 6.2: Summary simulation statistics for EOH-W and EOH-S

Although the differences in the storage CDFs are somewhat smaller than those for wealth, the differences do reflect the impact of the differences in relative preferences for EOH storage and wealth. Compared to W4S2, the W2S2 storage CDF is slightly more upright, indicating that lower storage levels are less desirable, and hence it is worthwhile having lower (worse) levels of EOH wealth in order to increase them.

When wealth has a higher relative risk aversion than storage, as for W4S2, a lower mean EOH wealth is acceptable if the variability in the EOH wealth outcomes is reduced. In order for this to be achieved, the variation in inflows is absorbed by storage, though the effect on mean EOH storage is variable, because a less variable EOH wealth implies less release on average, and hence higher storage, on average.

The functional form of the utility functions describes the preferences about EOH storage and wealth outcomes, as well as the relative preferences of each over the other. These preferences are reflected by changes in the mean and variance. The variance and quartile statistics reflect the observations made about the EOH wealth and storage CDFs. Compared to the RN case, the W4S2 and W2S2 EOH wealth quartiles tend towards the mean, which is less than the RN mean in both cases (see Figure 6.14). For W4S2, the low standard deviation of EOH wealth is paired with a higher standard deviation of EOH storage, though mean storage levels are higher than RN for both W4S2 and W2S2.

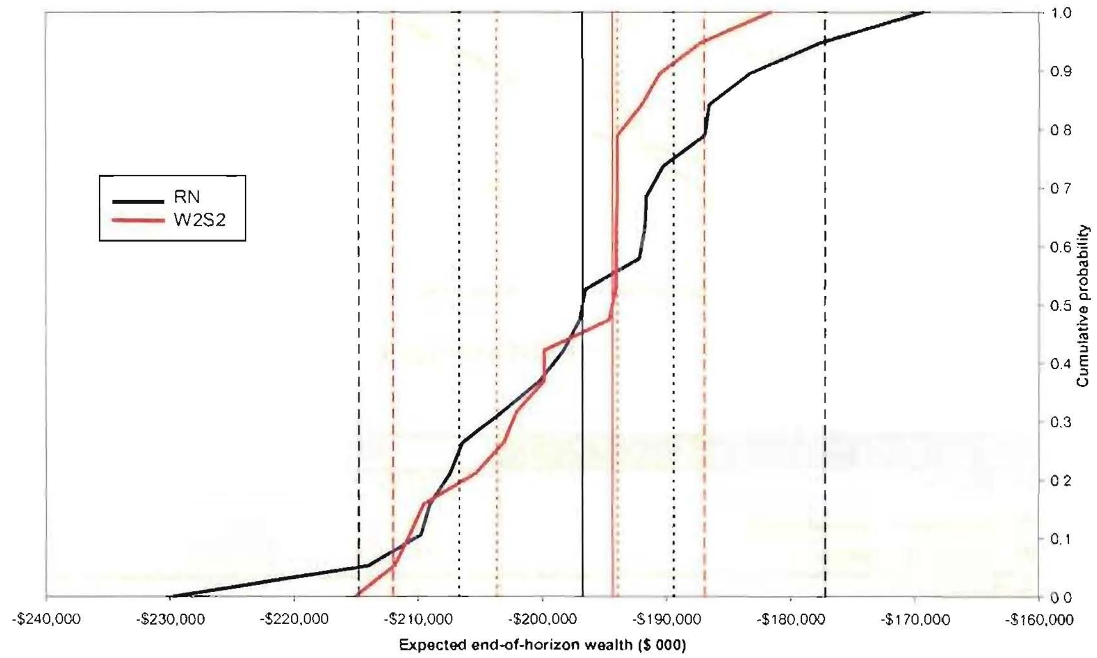


Figure 6.13: RN and W2S2 EOH wealth CDFs

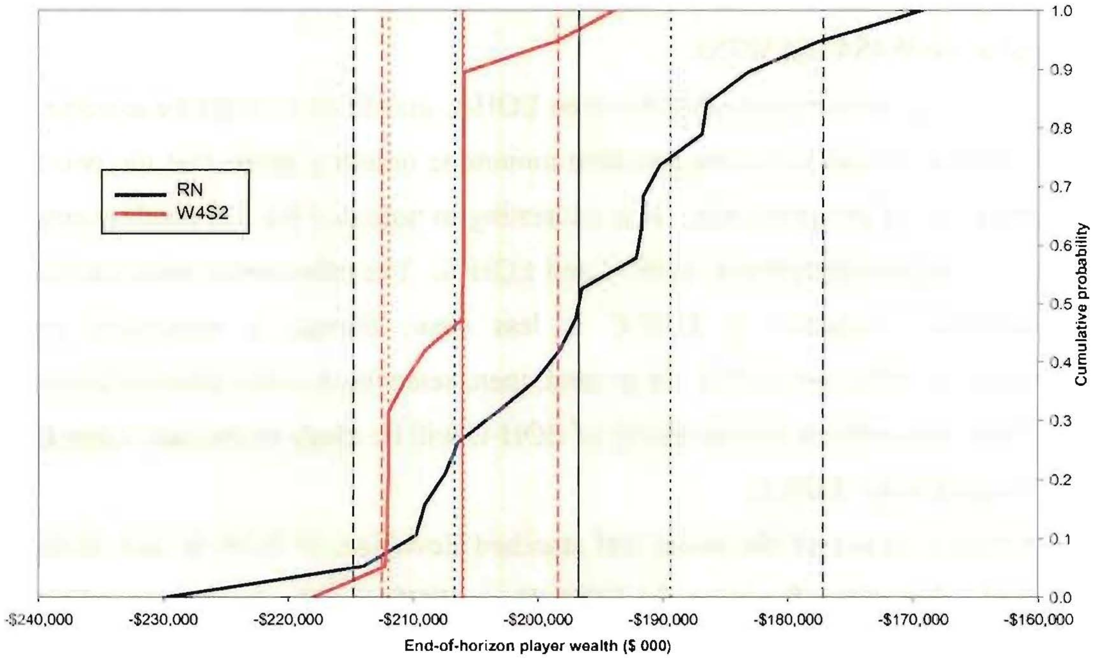


Figure 6.14: RN and W4S2 EOH wealth CDFs

To illustrate the impact of different preferences for EOH wealth and storage on the distributions of EOH wealth and storage, optimisation/simulations were performed for all combinations of U_W and U_S (ie, (W0,W1,W2,W3,W4)x(S0,S1,S2,S3,S4)). Combinations of means and standard deviations of EOH-W and EOH-S are plotted in Figure 6.15, Figure 6.16, and Figure 6.17. Note that in some figures the negative of EOH-W has been plotted on the x- axis, with these values being referred to as EOH-C (C=cost) i.e, EOH-C = -(EOH-W). Thus, 'better' EOH-C outcomes lie to the left of the figures rather than to the right, which is the case for EOH-W outcomes.

A positive and linear relationship (correlation coefficient = 0.996) appears to exist between mean EOH-C and mean EOH-S (Figure 6.15); an increase (worsening) in EOH-C is accompanied by an increase in mean EOH-S at a rate of approximately 0.044GWh/\$1000, or 44GWh/\$m. There are only two instances where a higher EOH-C has a lower mean EOH-S, and they occur at EOH-C values of approximately \$195m and \$200m. In both cases, the difference between the inconsistent mean EOH-S and the adjacent values is relatively small (≈20GWh). Also, the inconsistent results are achieved using utility functions with the same or similar utility functions applied to for EOH-W and EOH-S. At EOH-C=\$195m, the inconsistent point corresponds to W3S4, and the previous point corresponds to W1S2, while at EOH-C=$200m, the two functions are W4S4 and W3S3.

While a general relationship between EOH-C and EOH-S might be expected there is no a priori reason to expect a unique monotone ordering given that the two factors are being varied independently. It is interesting to note that the RN result is consistent with the relationship between EOH-C and EOH-S. The relationship between the mean and standard deviation of EOH-C is less clear, though is essentially negative (correlation coefficient=-0.93). In general, then, selecting a utility function (from those used here) that reduces the variability of EOH-C will be likely to increase mean EOH-C and increase mean EOH-S.

Extreme values of the mean and standard deviation of EOH-W and EOH-S are obtained using utility functions for EOH-W and EOH-S. The lowest mean EOH-C is obtained using W0S0, which, not surprisingly, also results in the lowest mean EOH-S and a relatively large standard deviation of EOH-C. It is one of five cases with an EOH-C standard deviation greater than $16m; all five cases use W0 (see also Figure 6.16). At the other extreme, W4S0 results in the highest mean EOH-S, the lowest standard deviation of EOH-C, but also the highest mean EOH-C.

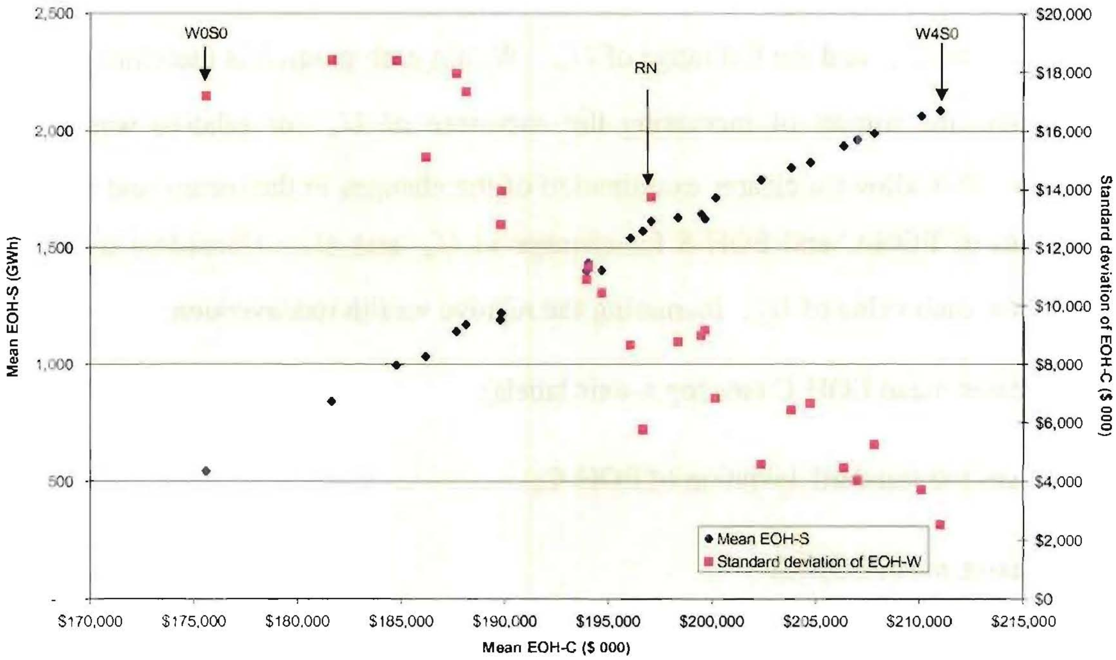


Figure 6.15: Mean EOH-C vs. mean & standard deviation of EOH-C (sorted by mean EOH-C)

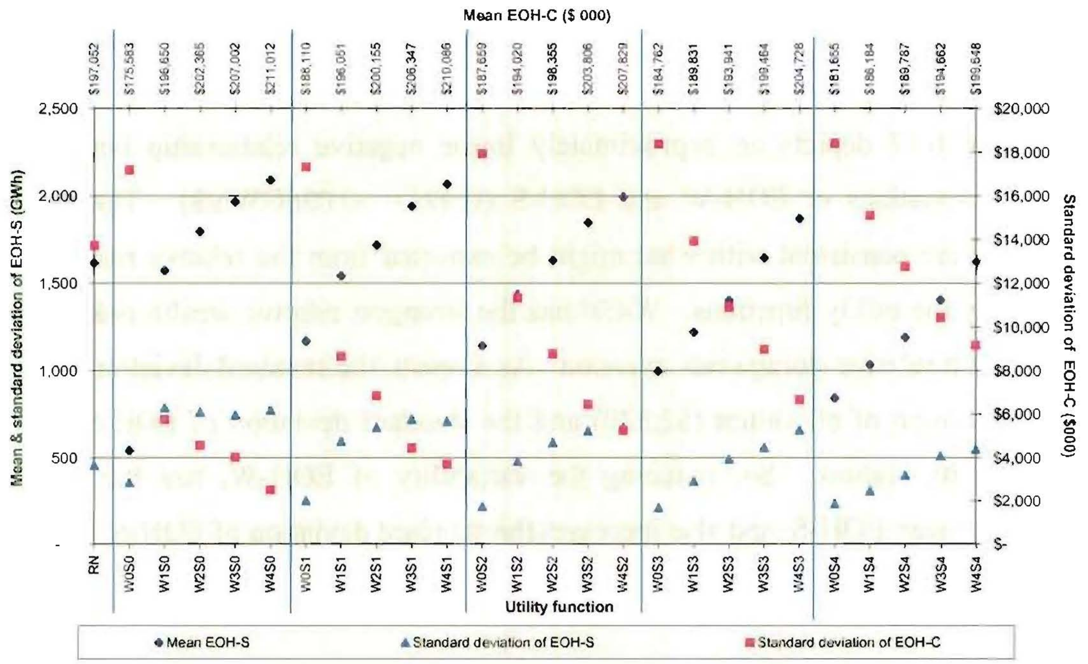


Figure 6.16: Mean and standard deviations of EOH-C and EOH-S

Figure 6.16 plots the same data as used in Figure 6.15 but ordered by utility function. The mean and standard deviation of EOH-S and standard deviation of EOH-C are then plotted for combinations of U_S and U_W, with the leftmost observation being the RN case, for reference. These results are collated into 5 groups with each group having a fixed U_S, and the full range of U_W. Within each group it is therefore possible to examine the impact of increasing the curvature of U_W, or relative wealth risk aversion. This allows a clearer examination of the changes in the means and standard deviations of EOH-C and EOH-S for changes in U_W and U_S. Consistent trends are evident for each value of U_S. Increasing the relative wealth risk aversion:

- increases mean EOH-C (see top x-axis labels);
- reduces the standard deviation of EOH-C;
- increases mean EOH-S;
- increases the standard deviation of EOH-S.

As the curvature in U_S increases, the groups of mean and standard deviation in EOH-S shift downward. In both instances, the curves become closer to linear as U_S increases. At the same time, the groups of EOH-C means and standard deviations shift upward, reflecting, again, the compensatory behaviour of the means and standard deviations of EOH-C and EOH-S.

Figure 6.17 depicts an approximately linear negative relationship between the standard deviations of EOH-W and EOH-S (slope = -0.036GWh/$). The standard deviations are consistent with what might be expected from the relative risk aversion implied by the utility functions. W4S0 has the strongest relative wealth risk aversion and weakest relative storage risk aversion. As a result, the standard deviation of EOH-W is the lowest of all values ($2,529) and the standard deviation of EOH-S (771) is almost at its highest. So, reducing the variability of EOH-W, has the effect of increasing mean EOH-S, and also increases the standard deviation of EOH-S.

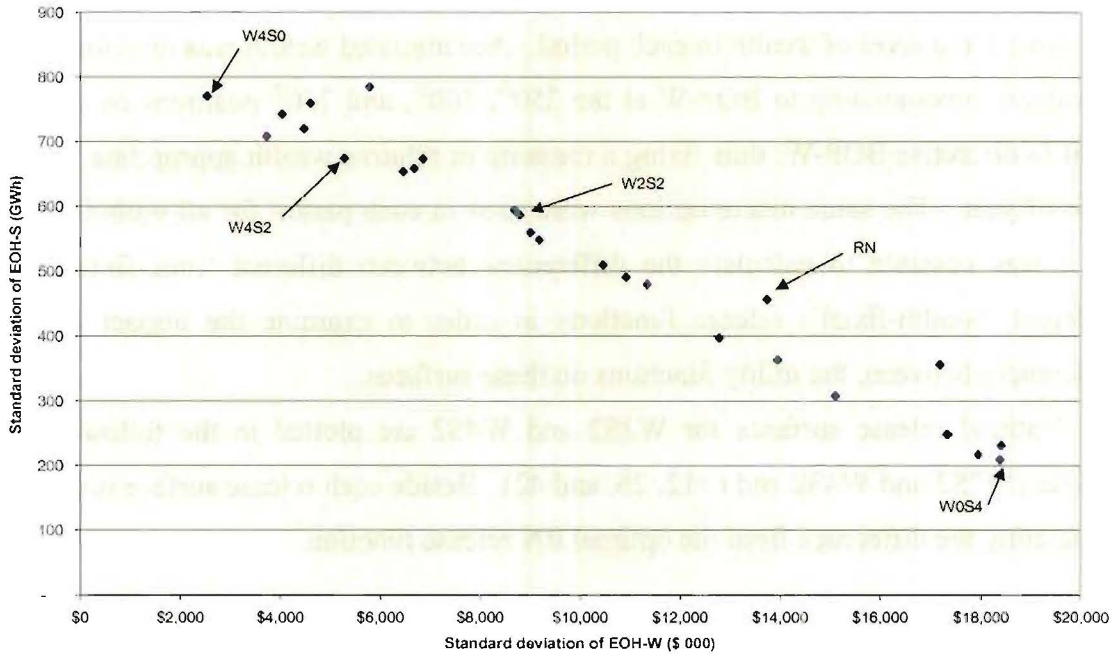


Figure 6.17: Standard deviation of EOH-W vs. standard deviation of EOH-S

6.5 Weekly operation

This section discusses the effects of the utility functions on the weekly operation of the reservoir, and the implied generation levels of the other stations in the system. The curvature in the end-of-period (EOP) cost-to-go function reflects the relative benefits of achieving different EOP-W and EOP-S outcomes. In Section 6.2, the optimal weekly RN release functions were illustrated (Figure 6.3), being only a function of storage. As discussed earlier, SUMDP uses a 2-dimensional state space to handle the bi-variate non-linear utility functions. For each period (stage) the release decision will therefore be a function of accumulated wealth and storage at the beginning of the period, referred to as BOP-W and BOP-S, respectively. t is therefore not possible to depict the weekly optimal release functions in a single figure because it is four-dimensional (time, wealth, storage, release).

Three dimensional surfaces were created by fixing either time or wealth. When time is fixed, the figures show the release function for a given period, and over the range of wealth and storage levels which can eventuate in that period. Figures were created for release in the 12th, 26th, and 42nd periods. When accumulated wealth is fixed, annual trajectories can be shown which reflect the range of releases in each week for the range of storage levels. Because wealth accumulates, it is not useful to define the same fixed level of wealth in each period. Accumulated wealth was therefore fixed at values corresponding to BOP-W at the 250th, 500th, and 750th positions on the grid used to discretise BOP-W, thus fixing a measure of relative wealth appropriate for that time of year. The same discretisations were used in each period for all optimisations, so it was possible to calculate the differences between different 'time-fixed' (and different 'wealth-fixed') release functions in order to examine the impact of, and differences between, the utility functions on these surfaces.

Optimal release surfaces for W2S2 and W4S2 are plotted in the following six figures (W2S2 and W4S2 and t=12, 26, and 42). Beside each release surface is a figure illustrating the difference from the optimal RN release function.

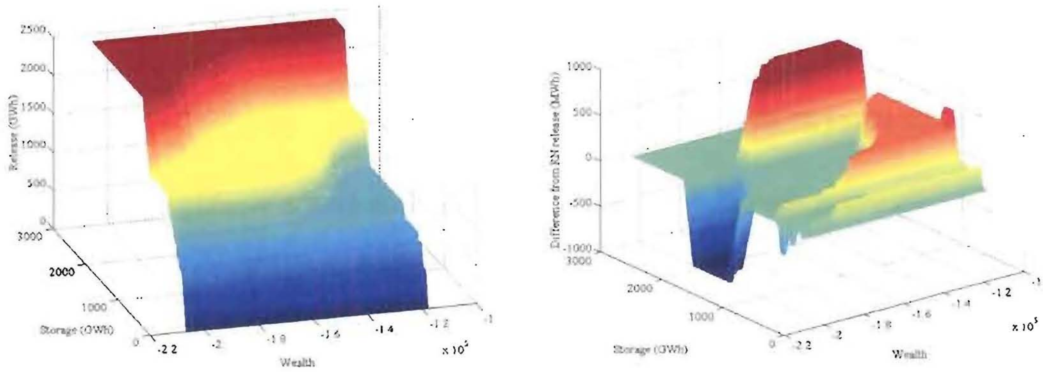


Figure 6.18: W2S2 optimal release surface and difference from RN release (t=42)

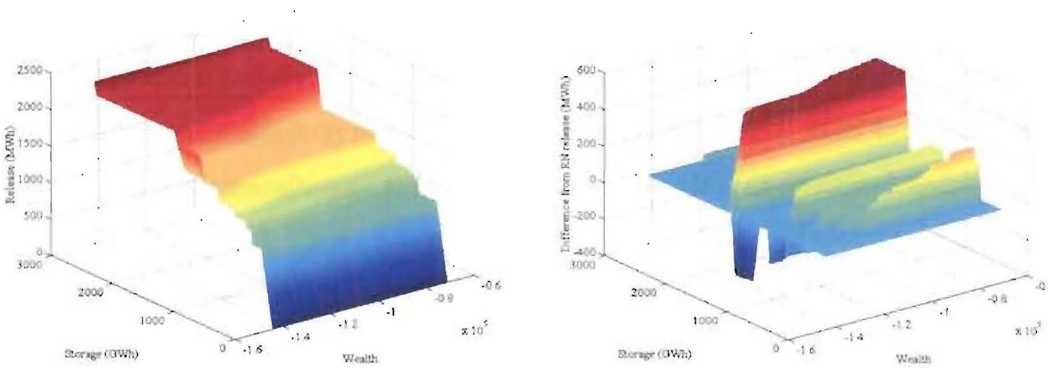


Figure 6.19: W2S2 optimal release surface and difference from RN release (t=26)

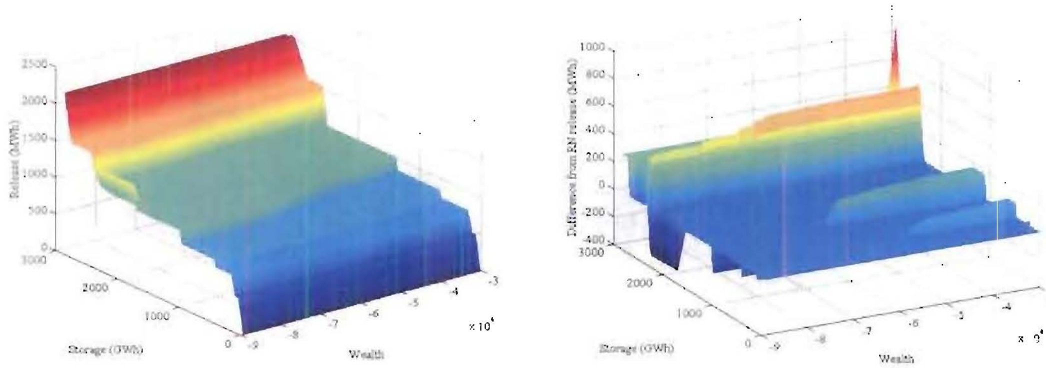


Figure 6.20: W2S2 optimal release surface and difference from RN release (t=12)

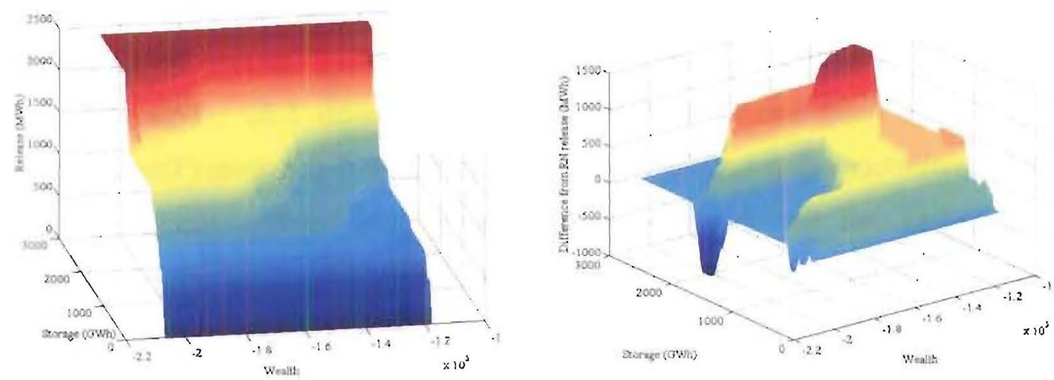


Figure 6.21: W4S2 optimal release surface and difference from RN release (t=42)

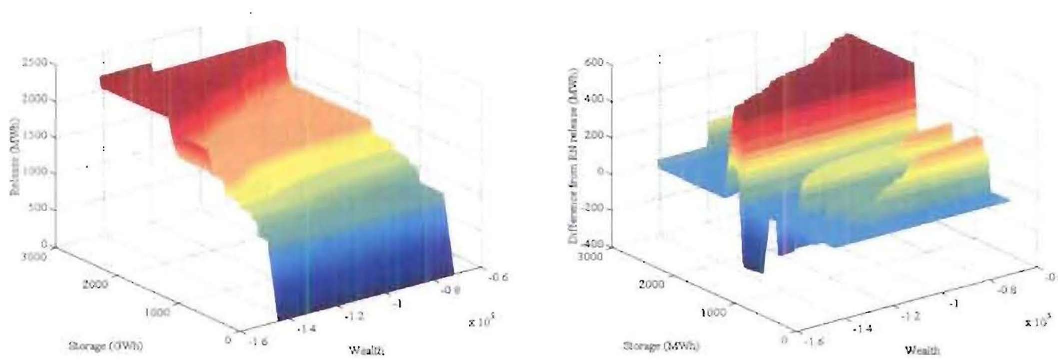


Figure 6.22: W4S2 optimal release surface and difference from RN release (t=26)

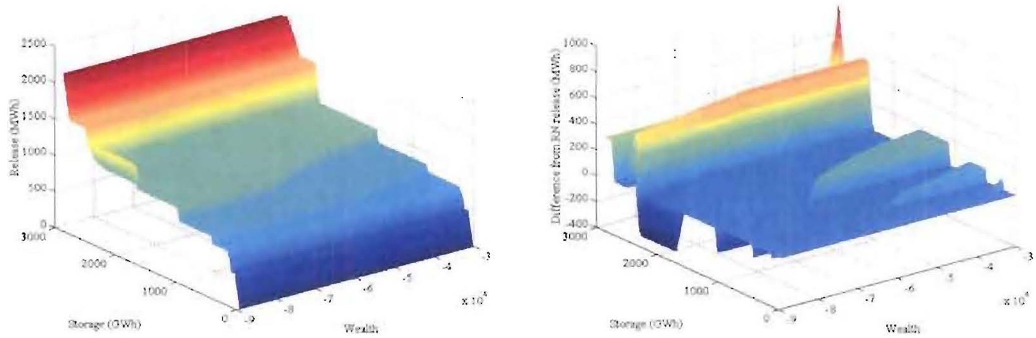


Figure 6.23: W4S2 optimal release surface and difference from RN release (t=12)

With the EOP value function non-linear in both EOP-W and EOP-S, the optimal release function will not necessarily be constant over any range of BOP-S and BOP-W. For fixed BOP-W, release increases as BOP-S increases (Theorem 5.2). For fixed BOP-S, release decreases as BOP-W increases (Theorem 5.3). Thus, the release function is non-decreasing as BOP-W decreases and BOP-S increases (Theorem 5.4).

The W2S2 and W4S2 release surfaces have 'flats' in the storage and wealth dimensions, which correspond to a single release being optimal for a range of BOP-W and BOP-W levels. As with the RN release function (Figure 6.3), these flats correspond to the intersection of a vertex of the STPC and a contour of the EOH value function. When a single release is optimal over a range of BOP-S and BOP-W, the slope of the EOH value function implies that an increase in release (and increase in wealth and decrease in storage) decreases expected EOH utility. Conversely, a 'sloped' section of the release surface corresponds to an edge of the STPC (a horizontal segment of the residual demand curve) intersecting a contour of the EOP value function. For t=42, the highest 'flat' corresponds to Huntly baseloaded, the next flat is New Plymouth baseloaded, and so on. For t=26, the highest flat corresponds to New Plymouth baseloaded, the next highest flat is Stratford baseloaded, and so on. For t=12, the highest flat corresponds to New Plymouth baseloaded, the next highest flat corresponds to Stratford baseloaded, and so on and so forth. (Where these flats overlap (for the two cases compared) the solutions are obviously identical, causing the erratic nature of the plot of differences).

When BOP-S is near its lower and upper bounds, release is the same regardless of the level of BOP-W. This implies that the marginal utility of storing another unit of water is essentially independent of the level of wealth. This is because the contours of the EOP value function are at their steepest when storage is low and flattest when storage is high. Therefore, the additional utility derived from increasing EOP-W is greater (less) than the decrease in utility from releasing another unit of water when storage is high (low). For BOP-S between the storage bounds, and across the range of BOP-W, the release functions exhibit variation, with the magnitude of change decreasing as the distance from the terminal period increases. This is a sensible and expected result, as well as a common characteristic of SDP solutions; decisions closer to the end of the horizon will be more sensitive to the form of the terminal value function than decisions further away. In period 42, for example, there is considerable curvature/variation in both the W4S2 and W2S2 release functions, while in period 12, there is comparatively little curvature/variation. This curvature reflects a transition from a 'low' release function when BOP-W is high to a 'high' release function when BOP-W is low.

The SUMDP release functions differ considerably from the RN release function, though the difference decreases for periods further away from the end of the planning horizon. The variations over the entire range of BOP-W and BOP-S are reflected in the figures above (adjacent to the optimal release surfaces). They show the difference between the RN release function and the SUMDP release functions for t=12, 26, and 42. A positive difference indicates that the RN release is higher than the corresponding W4S2 or W2S2 release. Consider the release functions with t=42 (Figure 6.18 and Figure 6.21):

- When BOP-W is low, optimal release is higher, indicating that New Plymouth would be marginal were BOP-S in the range ≈500-1500GWh (approximately). For storage levels >1500 GWh (approximately), release is higher, less thermal generation would be required, and so Huntly would be the marginal station.

- When BOP-W is high, optimal release is lower, indicating that in New Plymouth would be marginal for BOP-S in the range ≈2000-2500GWH; Stratford and Marsden-A would be marginal were BOP-S in the range ≈500-2000GWH.

Thus, the SUMDP release functions indicate that for a reasonable range of BOP-S, one of four stations could be marginal, depending on the level of BOP-W. This result contrasts with the optimal RN release function described in Section 6.2, which implied that New Plymouth was marginal for BOP-S in the range 279-2557GWh and that Huntly would only be marginal when storage levels were above 2500GWh.

Previous results showed that mean EOH-W was lower for the W2S2 and W4S2 cases compared to the RN case, with mean EOH-W decreasing as relative wealth aversion increased. In order for this lower EOH-W to be achieved, release must be lower. In the figures above, the optimal RN release is shown to be higher than the W2S2 and W4S2 optimal release for a large range of BOP-W and over the entire range of storage levels. This is indicated by a positive difference between the RN and SUMDP release functions. From these figures, and inspection of release functions with BOP-W fixed, the W4S2 release is generally less than the corresponding W2S2 release. This is reflected by the W4S2 release functions, for a particular level of BOP-W, placed to the right of the corresponding W2S2 release function. For relatively small ranges of BOP-W and BOP-S, though, the optimal RN release is less than the W4S2 and W2S2 optimal release. These ranges exist in the release surfaces of the three sample periods and increase, in terms of area and magnitude, as the time from the end of the horizon increases. Therefore the optimal release surface includes releases both greater and less than the RN optimal release, and this is reflected in the simulation results discussed later.

For periods 26 and 12, the release functions have a similar form to those for t=42, but with less curvature and variation. As a result, the figures showing the difference from the optimal RN release show a smaller deviation in terms of the magnitude as well as the range of BOP-S and BOP-W for which there is variation. Associated with this behaviour is an increase in the range of BOP-W and BOP-S for which there is a differential of zero, indicating an identical release to the RN release. This area is smallest for the period 42 results, and largest for the period 12 results.

Figure 6.24 shows the RN, W2S2 and W4S2 mean weekly release (per hour), taken from the simulation. The release profiles are not symmetric, although demand is symmetric. All three mean release curves have similar profiles in that they start at approximately 800MWh in the first week, increase and peak at approximately 2000MWh in the middle of the year, then decrease to about 1200MWH in the final week. Mean release is higher during the second half of the year. Presumably, this is because inflows are higher and increasing over that period and demand is decreasing.

During the first half of the year, inflows are lower and decreasing while demand is increasing. Demand peaks in the middle of the year, so higher releases are required in order to avoid the relatively large costs (and large decreases in wealth) incurred by operating more expensive thermal stations (recall that the starting storage level for the simulations was 1450GWh). Although the mean EOH-W for both W2S2 and W4S2 are lower than the RN EOH-W, mean release is not always lower than mean RN release. This is consistent with the release surfaces shown earlier.

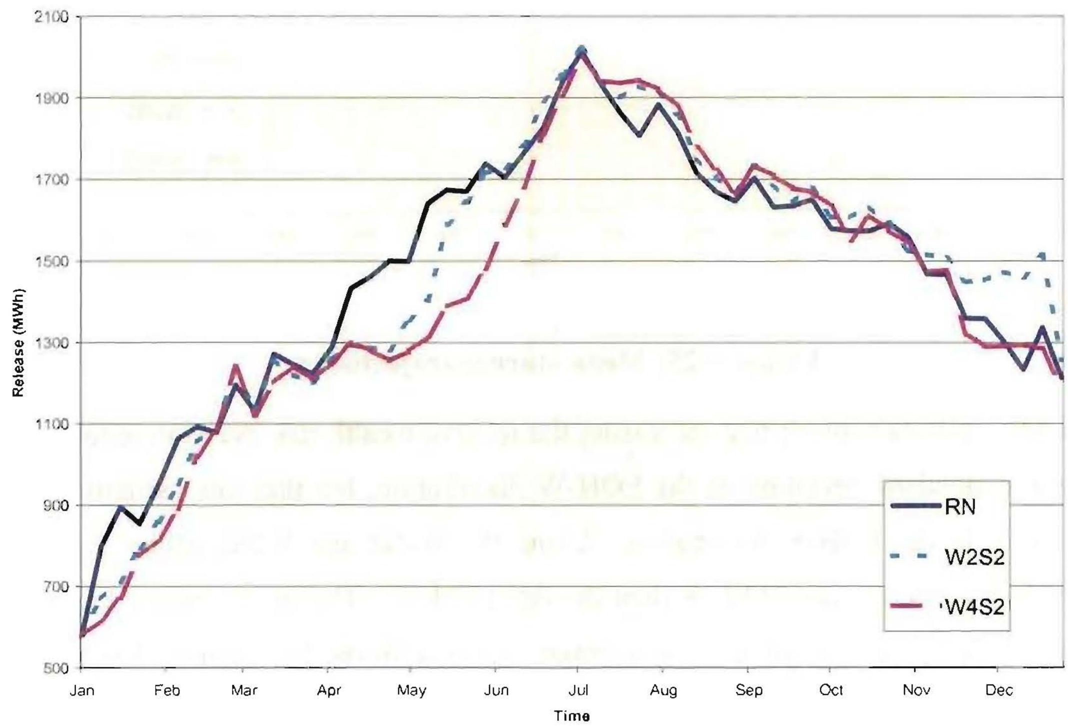


Figure 6.24: Mean weekly release

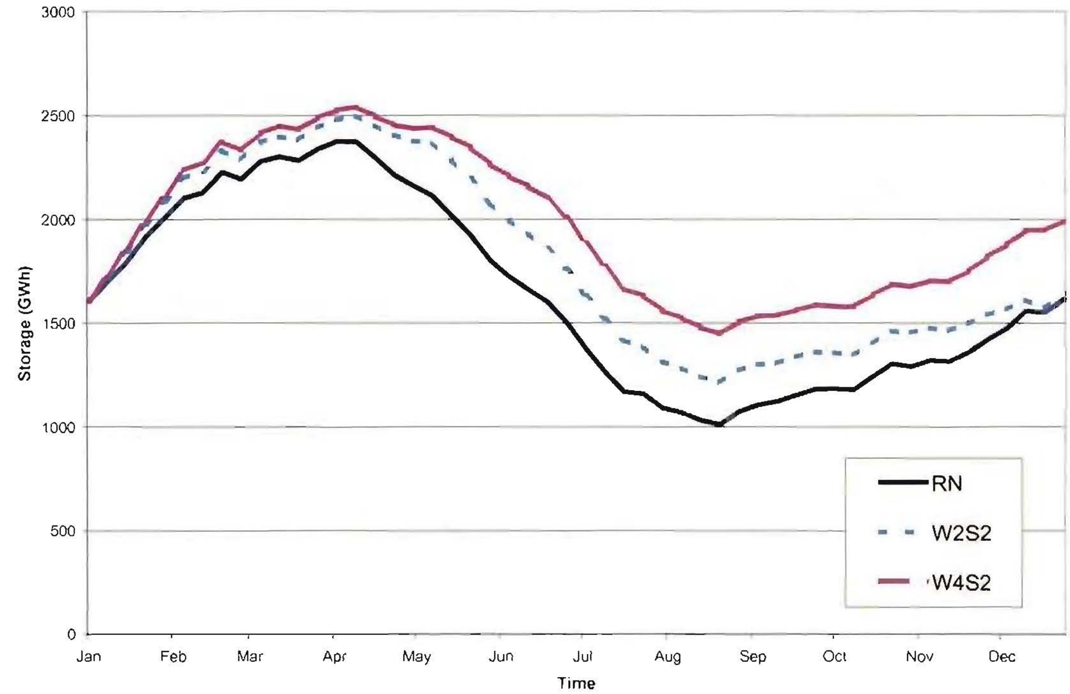


Figure 6.25: Mean storage trajectories

Earlier results showed that increasing the relative wealth risk aversion reduced the mean and standard deviation of the EOH-W distribution, but that this variability was transferred to the EOH-S distribution. Using the W4S2 and W2S2 utility functions resulted in a lower mean EOH-W than the RN EOH-W. The mean release and mean storage trajectories illustrate that, on average, this is achieved by releasing less (Figure 6.24) and storing more (Figure 6.25). Further, when relative wealth risk aversion is increased, relative storage risk aversion implicitly decreases, and it was shown earlier that the result is an increased mean and increased variation. Therefore, more water is stored for W4S2 than W2S2, which is reflected by the W2S2 release trajectory being higher, and the W2S2 mean storage trajectory being lower, than the corresponding W4S2 trajectories.

As an example of the impact on generation from sources other than the reservoir, consider Figure 6.26 and Figure 6.27, which show the average generation from the RN, and W2S2 and W4S2 simulations, respectively.

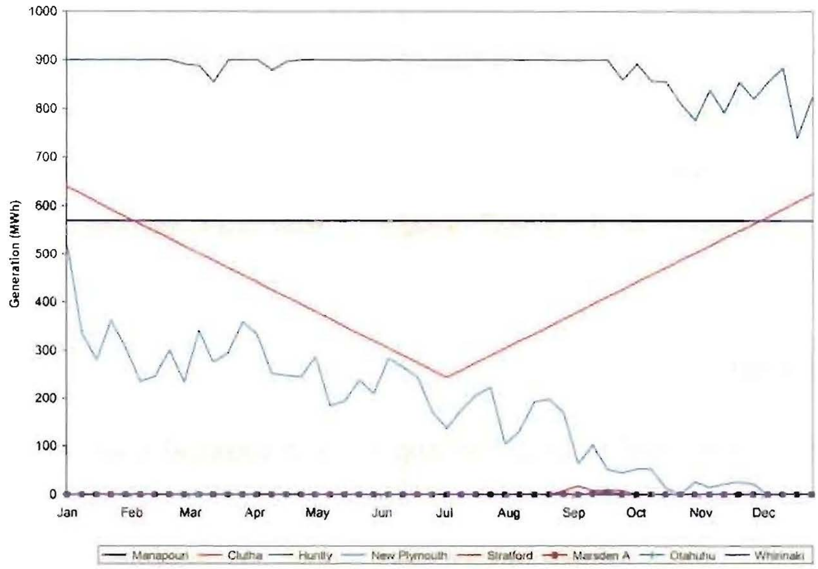
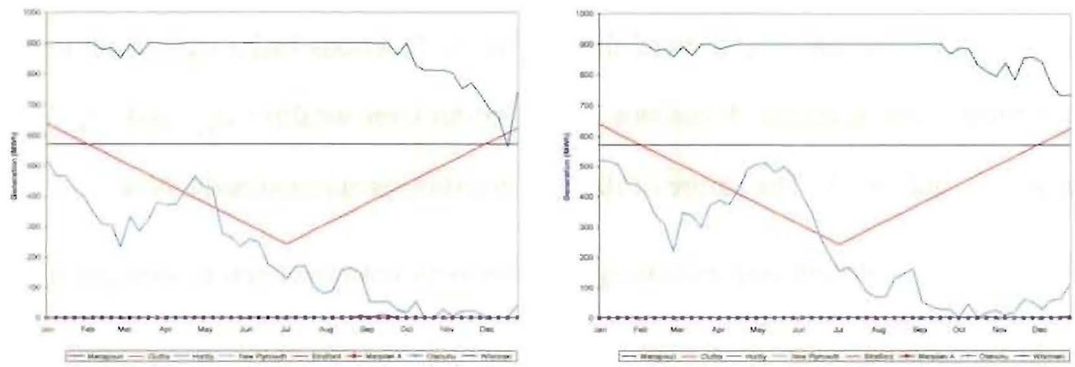


Figure 6.26: Mean weekly generation for 'other' stations (RN)



W2S2 W4S2

Figure 6.27: Mean weekly generation for 'other' stations

In all three cases, Manapouri and Clutha release at their maximum throughout the year. Huntly operates at its maximum through the middle of the year, when demand is higher, though operates at lower levels at the beginning and end of the year. All other stations have lower mean generation levels in the latter part of the year in response to the increased reservoir release from higher inflows. New Plymouth has the most variable generation in all three cases, with the shape of its trajectory dictated by the simulations all starting form the same storage level, and the increased reservoir release in the later half of the year. In Figure 6.24, the W4S2 and W2S2 mean release trajectories were lower then the RN trajectory during the April-July period, then slightly higher during the July-October period. The impact of this is to increase New Plymouth's mean generation in the first half of the year, enabling water to be conserved and used in the later half of the year and therefore minimise the extent to which New Plymouth and more expensive thermal stations would be required to generate over this period. In the RN case, Stratford, Marsden A, and even Otahuhu are dispatched in some simulations. These stations are used to a lesser extent for the W2S2 case (only Stratford used), and again for the W4S2 case, with Stratford operating in the last period in one simulation.

6.6 Conclusions

Results have been presented for SUMDP applied to a classical reservoir management problem where hydro release is used to offset thermal generation in order to meet demand. Rather than minimising expected cost, though, the total utility of accumulated cost (U_W) and storage (U_S) at the end of the horizon was maximised. An additive utility function was used, and comparing the results from using a variety of combinations of U_W and U_S showed that the utility functions had a significant impact on the means and standard deviations of end-of-horizon wealth (μ_{W_T} and σ_{W_T}) and storage (μ_{S_T} and σ_{S_T}). The nature of these interactions is summarised below:

- Holding U_W constant and increasing the concavity (risk aversion in storage) in U_S decreases μ_{S_T} and decreases σ_{S_T}.
- Holding U_S constant and increasing the concavity (risk aversion in wealth) in U_W decreases μ_{W_T} and decreases σ_{W_T}.
- An increase in μ_{W_T} (improvement) is accompanied by a decrease in μ_{S_T}.
- A decrease in σ_{S_T} is accompanied by an increase in σ_{W_T}.

In terms of system operation, the average storage and average release trajectories reflected the preferences implied by U_W and U_S. For example, if U_W had a higher degree of concavity than U_S, lower μ_{W_T} would be acceptable, and this would be achieved by releasing less, hence μ_{S_T} will be higher, though with a higher σ_{S_T}.

Compared to the risk neutral case, risk aversion produced a more conservative release strategy in the first part of the year and a more aggressive strategy in the latter part of the year. As a result, generation from higher cost thermal plant was shifted to the first part of the year to meet demand, depending on the extent to which high cost outcomes are to be avoided, as implied by the utility function.

Overall, the experimental results detailed here indicate that risk aversion, as reflected by a (non-linear) utility function can imply release strategies which have a significant impact on the distributions of expected end-of-horizon wealth and storage. In the next chapter, these effects are investigated for a reservoir operated in a 'deregulated' electricity market.

Chapter 7

SUMDP and Reservoir Management in a Deregulated Electricity Market

7.1 Introduction

In Chapter 5, the SUMDP model was applied to the operation of a single reservoir in a 'regulated' environment where a single entity manages the reservoir and multiple thermal stations to meet demand. The decision variable in each period (week) was the release, or generation, from the reservoir, which implied the (deterministic) quantity of thermal generation required to meet demand. The cost of the thermal generation was the weekly return in the SDP algorithm. The accumulated weekly costs and the terminal storage were then used as the argument for the terminal value function, which was the utility associated with various combinations of the two attributes.

In the regulated case, then, the reservoir and other generating plant were coordinated so as to meet demand. The accumulated cost of satisfying demand not met by reservoir release was the argument of the utility function. The reservoir was therefore used in a conventional manner i.e., as a means of alleviating the cost of running expensive thermal plant. In a deregulated electricity system, the release variable can represent the firm's interaction with an electricity market in a given period. The obvious candidate for the return from release is profit, and this is used here.

In order to determine the profit from release in a given period, the behaviour of the other firms in the market must be represented in the model. A simple modification to the benefit function described in means that a 'dominant firm' scenario can be modelled, with the reservoir firm being dominant and the remaining firms acting as a fringe of price takers, even though they may set the price. In this model, each fringe firm offers their entire capacity to the market at marginal cost and will generate the quantity required to clear the market, so long as the spot price is greater than their marginal cost of generation. The reservoir manager has perfect knowledge of demand and the fringe capacities and costs when making the release decision in each period. Given some level of reservoir release, the fringe is just dispatched in a merit order of marginal cost so as to meet the residual demand. In a given period, the market price is set by the most expensive fringe station, which then determines the revenue received by the hydro firm from its release. The price associated with the release is used to create the benefit curve used as an input to the SUMDP optimisation. The objective is to maximise the firm's expected utility of end-of-horizon wealth (W) and the end-of-horizon storage.

Although the hydro firm controls the market price and hence possesses a high degree of market power; prices may not be pushed as high as in a Cournot model because the fringe offer their capacity at marginal cost. On the other hand, the reservoir firm has more control over its market share. This is a different representation of the nature of competition than the Cournot models of Scott (1997) and Bushnell (1998) which have been applied in a reservoir management context. (As a further extension, a Stackelberg leader/follower model is described in). Rather than the other firms behaving as perfect competitors, a Cournot model is used to represent competition between the fringe firms for the demand not satisfied by reservoir release).

The firm is assumed to be contracted for a fixed quantity of electricity in each week. The contract quantity and strike price are input parameters of the model, and the other competitors are not contracted. Batstone and Scott (1998) examine the case where the spot market is a Cournot game and all the players can be contracted, though they assume the players are risk neutral towards wealth.

In this chapter, a variation of the previous model is introduced to address the case where a dominant and price-setting hydro firm operates a single reservoir in an electricity market. This case is referred to as the deregulated case. Section 7.2 describes the formulation. The benefit function and a solution algorithm are discussed in Section 7.3, with conclusions in Section 7.4. Experimental results using this model are presented and discussed in and some alternative benefit functions are presented in .

7.2 SUMDP and 'deregulated' reservoir management

The formulation is essentially the same as described in , though the weekly return function now reflects the benefit to an individual firm rather than the system as a whole. We assume that the firm controls the weekly reservoir release (generation) made in each week. Reservoir storage and release are bounded and the inflows experienced in each week are uncertain. The relationship between q^t and s^{t+1} is assumed to be constant and negative and is not affected by factors such as the form of the market or storage levels (head effects). Demand must be met in each period and there are bounds on release and on storage. The inflows experienced in each week are uncertain. The formulation is the same as that presented in , but is repeated here for ease of reference.

$$f^t\left(w^t, s^t\right) = \max_{\underline{q}^t \le q^t \le \overline{q}^t} \mathrm{E}\left[f^{t+1}\left(w^{t+1}, s^{t+1}\right)\right] \qquad \forall t$$

subject to:

$$w^{t+1} = w^t + B^t\left(q^t\right) \qquad \forall t$$

$$s^{t+1} = s^t - q^t + a^t \qquad \forall t$$

$$\underline{q}^t \le q^t \le \overline{q}^t \qquad \forall t$$

$$\underline{s}^t \le s^t \le \overline{s}^t \qquad \forall t$$

$$w^1 = 0 \qquad (7.1)$$

where

T is the number of periods (t) in the planning horizon.

$f^t(w^t, s^t)$ is the expected end-of-horizon utility for a given wealth and storage combination in period t.

w^t is the firm's accumulated profit at the beginning of t.

s^t is the reservoir storage level at the beginning of t (measured in units of electricity e.g. MWh).

a^t is the stochastic inflow (in MWh) into the reservoir during t.

q^t is the release (in MWh) during t. The optimal release in a given period, $\hat{q}^t(w^t, s^t)$, is conditional on w^t and s^t i.e., $\hat{q}^t(w^t, s^t) = \underset{\underline{q}^t \le q^t \le \overline{q}^t}{\arg\max}\ \mathrm{E}\left[f^{t+1}(w^{t+1}, s^{t+1})\right]$.

$B^t(q^t)$ is a function describing the benefit (or return) from release. The benefit function is different in each period because it incorporates parameters such as demand, contracts, and the behaviour of other firms.

$\underline{s}^t, \overline{s}^t$ are the lower and upper bounds on storage in t.

$\underline{q}^t, \overline{q}^t$ are the lower and upper bounds on release in t. We assume that q^t is made before a^t is known, so a feasible release also satisfies $q^t \le (s^t - \underline{s}^t)$. Equivalently, the minimum release bound can be represented by $\overline{q}^t$ being a non-decreasing function of the storage level and $q^t \le \overline{q}^t(s^t)$.

While the formulation and basic solution approach are the same as described for the regulated case (), the benefit function $B^t(q^t)$ is redefined so as to reflect the returns from reservoir release in an electricity market. Consequently, the definition and implications of the release variable are different. (Other variations could be modelled by varying and/or extending components of the basic formulation while keeping the basic dynamics the same; a few of these are discussed further in).

7.3 The benefit function

The market is represented as the hydro firm and an additional I price-taking firms which are not expected to attempt to influence the price in the market. In each period they will generate as much as possible as long as the price is greater than in the marginal cost of generation (c_i), and are referred to as the 'fringe'. The time resolution of the model is a week. Weekly demand for electricity is assumed to be fixed at quantity g_0^t, which does not vary as the price changes. Thus, the demand curve in t is

$$d^t(p) = g_0^t \tag{7.2}$$

It is assumed that any level of q^t (measured in MWh) feasible to the hydro firm can be sold to the market. Let g_i be firm i's generation level, with total fringe generation being $g_f = \sum_{i=1}^{I} g_i$. The fringe is assumed to satisfy the residual demand by dispatching stations in order of marginal cost until $g_f^t = d^t(p) - q^t$. Equivalently, the residual demand curve faced by the hydro firm can be created by subtracting the fringe supply curve from the demand curve.

The residual demand curve faced by the hydro firm can therefore be defined as

$$d_r^t(p) = d^t(p) - S_f^t(p) \tag{7.3}$$

where $S_f^t(p)$ is the fringe supply curve. $S_f^t(p)$ is a monotonically increasing function created by adding the quantities and marginal costs of the fringe firms in order of marginal cost. The spot price that will result from release q^t can then be read off the inverse of $d_r^t(p)$. The benefit function, $B^t(q^t)$, is created by calculating the revenue from release given the residual demand curve and the firm's contract level, f^t. Revenue from release is then defined as the sum of contract revenue and spot revenue in the period:

$$B^t(q^t) = p_F^t f^t + p_S^t(q^t - f^t) \tag{7.4}$$

where p_F^t is the contract strike price and p_S^t is the spot price. The objective under consideration here is not revenue maximisation but utility maximisation, and utility is based on total revenues earned throughout the planning horizon. The relevant characteristics of contracts for the context described here are when the contract is sold, when the electricity (or contract quantity) is delivered, and when the revenue is received. In this model, the contracts are assumed to have been sold prior to the planning horizon, so that quantities are fixed in each period of the planning horizon (though can vary over t). These levels could be derived from a higher level model. (Ideally, f^t and p_F^t would be variables, though modelling contracts dynamically has its own complexities which are outside the scope of this work and certainly a topical research area (see Ranatunga (1995), Fleten et al (1999), or Batstone (2002) for research in this area)). A similar approach to that of Scott (1997) is used here where the contract level is varied assesses their impact on system performance.

The contract revenue is received at the time of delivery, so the revenue in t is $p_F^t f^t$, with p_F^t also fixed. If the firm were risk neutral, such that $U(\bullet)$ is linear, then this constant contract revenue could be ignored because

$$E\left[U\left(\sum_t p_F^t f^t + \sum_t p_S^t\left(q^t - f^t\right)\right)\right] = E\left[U\left(\sum_t p_F^t f^t\right)\right] + E\left[U\left(\sum_t p_S^t\left(q^t - f^t\right)\right)\right] \quad (7.5)$$

With $U(\bullet)$ non-linear, contract revenue can not be separated out from the difference payments because the utility of the contract and spot revenues is not additive. Note, though, that by defining utility over end-of-horizon profit, the timing of the contract payments is assumed to be irrelevant.

The impact on $B^t\left(q^t\right)$ of assuming a fixed demand curve and stepped supply curve is that it is not continuous over the range of release values and has a saw-tooth shape (though for ease of illustration, these segments are sometimes connected). Increasing q^t can change p_S^t, and p_S^t is applied to the entire quantity of generation; the firm has monopoly power on the spot price.

Figure 7.1 illustrates a weekly benefit curve when demand is fixed and the contract price is zero (p_F^t=0).

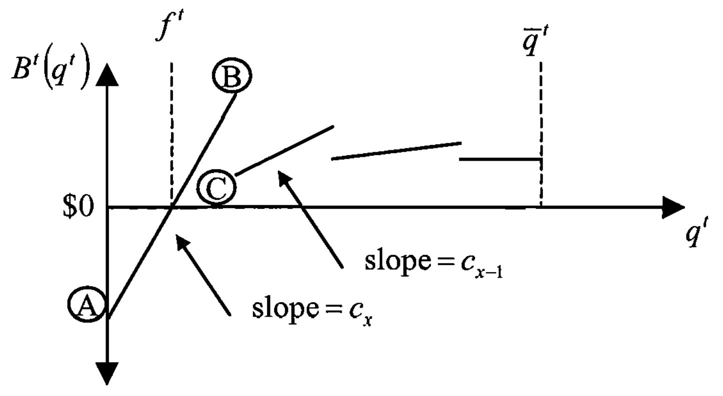


Figure 7.1: Benefit curve

If station x is marginal in period t for some release q', then increasing release to $q' + \Delta$ will result in either x remaining marginal, and the spot price remaining at c_x, or x being removed from the dispatch and replaced by station *x-1*, with marginal cost $c_{x-1} < c_x$. The lower marginal cost of station *x-1* is reflected by the flatter slope of the benefit function. From point 'A', then, as q^t increases and while station x remains in the dispatch, $p_S^t = c_x$. Total wealth, $\overline{w}^{t+1}$, increases linearly at the rate of c_x up to the point at which x is generating at its minimum (point 'B'). As release is increased further, the next cheapest station becomes marginal and $p_S^t = c_{x-1}$ (point 'C'). The impact of this on the benefit calculation is that the revenue from release decreases because p_S applies to all q_k^t and $c_{x-1} < c_x$. As release increases, the fringe continues to be displaced from the dispatch and the spot price decreases.

If $q^t < f^t$ the firm has to 'buy back' the difference between its release and the contract quantity at the spot price, and the firm's spot revenue will be negative. If $q^t > f^t$ then $p_S^t\left(q^t - f^t\right)$ is positive and wealth will increase. As the contract quantity increases, the benefit curve 'skews' in a clockwise direction such that $q^t < f^t$ becomes increasingly less costly and $q^t > f^t$ becomes increasingly less profitable, so the firm has less ability, and incentive, to manipulate the spot price. The effect of a non-zero p_F^t is to shift $B^t\left(q^t\right)$ vertically by the quantity $p_F^t f^t$. The larger the contract revenue component in $B^t\left(q^t\right)$, the lower the impact of changes in spot revenue on marginal benefit.

7.3.1 State transition

Consider now the process of determining the release that maximises the end-of-horizon expected utility. The transitions of (w^t, s^t) on to $W^{t+1} \times S^{t+1}$ depend upon the release, demand, contract level, and inflows in the period. The wealth and storage state spaces are discretised at $m=1 \ldots M$ and $n=1 \ldots N$ points, respectively. The values of w^{t+1} and s^{t+1}, for a particular w_m^t, s_n^t, and q_k^t, are denoted by $\overline{w}_{m,n,k}^{t+1}$ and $\overline{s}_{m,n,k}^{t+1}$, respectively.

A state transition possibility curve for a (w_m^t, s_n^t) pair is illustrated in Figure 7.2, with $B^t(q^t)$ mapped on to $W^{t+1} \times S^{t+1}$. The first point on the state transition possibility curve is point 'A', corresponding to release $\underline{q}^t$, which in this case equals 0MWh. If p_S^t and f^t are positive, demand is met wholly by fringe, with the marginal station (x) having marginal cost c_x. With $f^t > 0$, the firm must buy back the quantity $(f^t - 0)$MW at price p_S^t $(=c_x)$ and incurs (negative) spot profit $p_S^t f^t$. The point at which a positive spot profit is made can be found by drawing a vertical line through the point at which (w_2^t, s_2^t) maps on to $W^{t+1} \times S^{t+1}$.

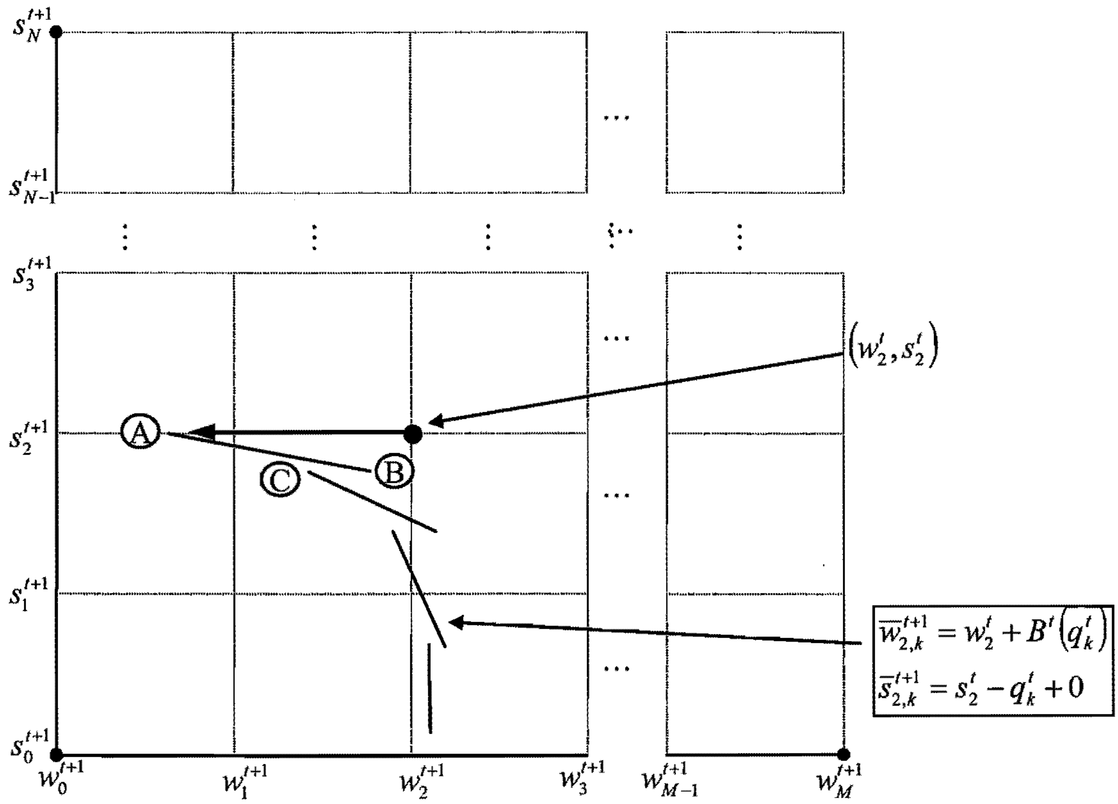


Figure 7.2: State transition possibility curve

There is no adjustment made here for the inflows. For reasons of computational efficiency, the value surface is adjusted for the inflow uncertainty prior to the calculation of $(\overline{W}^{t+1}, \overline{S}^{t+1})$ and the selection of the optimal release levels. As with the deregulated case, the inflow is an independent stochastic variable so $f^{t+1}(W^{t+1}, S^{t+1})$ can be adjusted for inflows, with the resulting inflow-adjusted expected utility function referred to as $g^t(W^{t+1}, S^{t+1})$.

Recall that, in a given period, *m* and *n* are the indices for the discrete levels of wealth and storage and *k* is the index for the set of discrete release levels. The optimal release for a point in $W^t \times S^t$, $\hat{q}^t(w_m^t, s_n^t)$, maximises $g^t(\overline{w}^{t+1}, \overline{s}^{t+1})$ i.e.,

$$\hat{q}^t(w_m^t, s_n^t) = \arg\max_{q_k^t} g^t(\overline{w}_{m,k}^{t+1}, \overline{s}_{n,k}^{t+1}) \qquad \forall k \qquad (7.6)$$

where $\overline{w}_{m,k}^{t+1}$ and $\overline{s}_{n,k}^{t+1}$ are calculated as

$$\overline{w}_{m,k}^{t+1} = w_m^t + B^t(q_k^t) \qquad (7.7)$$

$$\overline{s}_{n,k}^{t+1} = s_n^t - q_k^t \tag{7.8}$$

The expected utility at the beginning of t for a particular $\left(w_m^t, s_n^t\right)$ pair is the value of $g^t\left(\overline{w}^{t+1}, \overline{s}^{t+1}\right)$ corresponding to $\hat{q}^t\left(w_m^t, s_n^t\right)$, or

$$f^t\left(w_m^t, s_n^t\right) = \max_{q_k^t} g^t\left(\overline{w}_{m,k}^{t+1}, \overline{s}_{n,k}^{t+1}\right) \qquad \forall k \tag{7.9}$$

When $\overline{w}_{m,k}^{t+1}$ and $\overline{s}_{n,k}^{t+1}$ do not fall on points in $W^t \times S^t$, $f^t\left(w_m^t, s_n^t\right)$ is calculated by interpolating between the values of $g^t\left(w^{t+1}, s^{t+1}\right)$ corresponding to the discrete wealth and storage values that surround ($\overline{w}_{m,k}^{t+1}, \overline{s}_{n,k}^{t+1}$), as in .

The computational effort required to determine $f^t\left(w_m^t, s_n^t\right)$ over $\left(w^t \in W^t, s^t \in S^t\right)$ is dependent on the form of the water value function, the benefit function, and the end-of-horizon utility function. In the regulated case () significant computational savings (approximately 99%) were achieved because the search for the optimal release could be 'ratcheted'. This was possible because the benefit function was non-decreasing as storage decreased, independent of the level of storage and wealth, and the end-of-horizon utility function was concave in both storage and wealth. The state transition possibility curve for the revenue maximiser is not necessarily non-decreasing for decreasing storage, so the schemes described in can not necessarily be implemented in the form they are described. This issue is discussed in the next section.

7.3.2 Implementation issues

A less efficient implementation of the ratchet scheme described in is still possible if demand is assumed to be fixed in the period. A solution algorithm is as follows:

Determine $f^{T+1}\left(w_m^T, s_n^T\right)$
Create $g^t\left(w^{t+1}, s^{t+1}\right)$
$rr_n = 0 \ \forall n$
For $t = T \ldots 1$ (period)
  For $m = M \ldots 1$ (wealth w_m^t)
    For $n = 1 \ldots N$ (storage s_n^t)
      $f^t\left(w_m^t, s_n^t\right) = -\infty$
      $\bar{k} = \max\left(rr_n, rr_{n-1}\right)$
      For $p = 1 \ldots P$ (segment of $B^t\left(q_k^t\right)$)
        For $k = \underline{k}(p) \ldots \bar{k}(p)$ given $q_k^t < s_n^t$ (release q_k^t)
          $w^{t+1} = w_m^t + B^t\left(q_k^t\right)$
          $s^{t+1} = s_n^t - q_k^t$.
          $f_{m,n,k}^t = g^t\left(w^{t+1}, s^{t+1}\right)$
          If $\bar{f}_{m,n,k}^t > f^t\left(w_m^t, s_n^t\right)$
            $f^t\left(w_m^t, s_n^t\right) = \bar{f}_{m,n,k}^t$
            $\hat{q}^t\left(w_m^t, s_n^t\right) = q_k^t$
            $rr_n = k$
          else
            $k = \bar{k}(p) + 1$
          end else
        end for k
      end for p
    end for n
  end for m
end for t

Figure 7.3: Solution algorithm with release ratchets for price setting reservoir

For the range of q^t for which a particular thermal station is marginal, the slope of the benefit function is constant and hence is uni-modal. Therefore we can treat each segment of the state transition curve (where $p_S^t = \pi_x$) as a separate benefit function and apply the same techniques used when the benefit function is uni-modal over $\underline{q}^t \le q^t \le \bar{q}^t$. The overheads of this scheme would justify its use for thermal stations with a relatively large generation capacity, but it may well be quicker to search the entire range of generation levels for small stations.

7.4 Conclusions

This chapter has described the application of SUMDP to a situation where a reservoir firm is a price setter in an electricity market. Whereas the benefit in each period for the deregulated case was the cost of satisfying demand, for this deregulated case it was defined as the profit from hydro release and took into account the impact of an assumed contract quantity and strike price. The next chapter details experiments performed for this case, with utility defined over the accumulated profit and storage at the end of the horizon.

Chapter 8

Experimental Results for Deregulated Reservoir Management

8.1 Introduction

In this chapter, SUMDP is used to optimise release for a single hydro firm ("the firm") operating in a wholesale electricity market setting[2]. The market is assumed to consist of the hydro firm and five other firms (SOE1, SOE2, Contact, Transalta, & Other) and a notional 'Shortage' firm[3]. Excepting the hydro firm, each firm owns at least one generating station, with the capacities and marginal costs of each station used to create a supply curve in each period.

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[2] This example is loosely based on historical data. While firm names and generic assumptions from the New Zealand system have been used here, they are not necessarily representative of the New Zealand system as it operates in reality. The reader should not draw any conclusions from results detailed in this chapter with respect to the performance of the corresponding firms in the New Zealand electricity market, under present conditions.

[3] For the experiments performed here, 'shortage' was never called upon, which is a somewhat pleasing outcome.

Firm	Station	Capacity (MW)	Price ($/MWh)
SOE1	Waikato	1038	2.5
SOE2	TPD	335	1
	Huntly unit 1	230	2
	Huntly unit 4	250	4.62
	Huntly units 2&3	500	26.69
	Te_Awamutu	25	41.45
Contact	Wairakei & Ohaaki	208	0.5
	Clutha	639/244	1
	OtaCC	336	3.38
	NPL	230	4.9
Transalta	S_down	100	1.5
	TCC	379	3.39
Other	Geo	114	0.5
	NI Hyd	278	1
	SI Hyd	62	1
	Cogen	150	1.5
	Kiwi	45	33.94
Shortage		∞	500

Table 8.1: Station data

Thermal and geothermal stations had the same price and capacity in all periods. Using historical inflow data, the weekly capacity for each run-of-river station was set to the average tributary flow in the week. The hydrological aspects of all hydro stations aside from the firm's were not explicitly modelled. The weekly capacity for SOE1 was defined as its average weekly generation. This data is detailed in Appendix 3. The original data set had several stations with undefined or zero marginal costs (e.g. all run-of-river hydro). To ensure non-zero spot prices in situations where run-of-river units were marginal, it was necessary to set non-zero marginal costs for all stations. These marginal costs were also used in profit calculations for those firms.

Demand is fixed in each week, corresponding to a single block approximation to the LDC in each week (annual demand is 34.9GWh). Figure 8.1 shows the annual demand profile. The large drop in demand in the middle of the profile corresponds to the Christmas period.

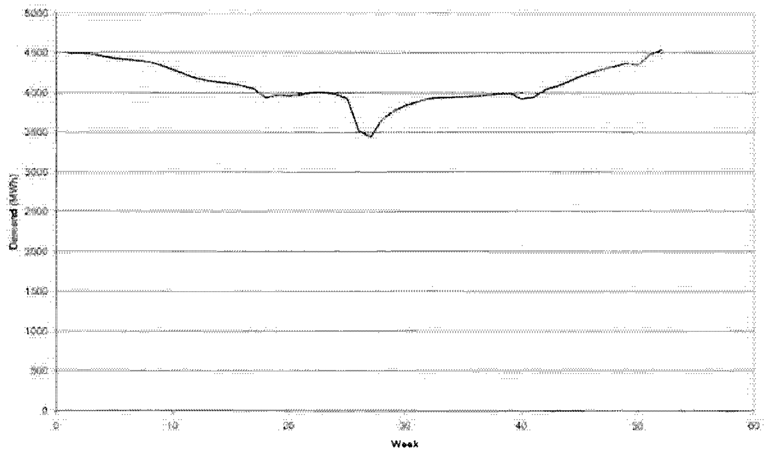


Figure 8.1: Demand profile

The planning horizon began in July. Maximum storage for the firm was 2600GWh. Maximum release in an hour was 2344MWh (393.8GWh/week) and minimum release set to the level of average tributary inflows. In addition to the market supply and demand curves, the contract level (f'), and the strike price (p'_F) were parameters manually set in each period. Contracts were defined as a fixed percentage of the firm's release capacity. For example, a 50% contract level corresponded to a contract quantity of 1172MWh/hour (196.9GWh/week) which is approximately 8% of the firm's storage capacity. More importantly, since average capacity utilisation is significantly less than 100%, a "50%" contract level corresponds to approximately 114% of average annual generation. Contracts were not modelled for any of the other firms, but are effectively assumed to be "reasonable" inasmuch as the behaviour of those firms is assumed to be represented by stable offers, as outlined above[4].

The starting storage level for each simulation was 2080GWh (=80% of capacity). Figure 8.2 shows the mean and standard deviation of weekly inflows. The raw inflow and tributary data is detailed in Appendix 3.

[4] Although those offers should not necessarily be interpreted as marginal costs, this stable behaviour at least implies that those firms are not opportunistically exploiting market power.

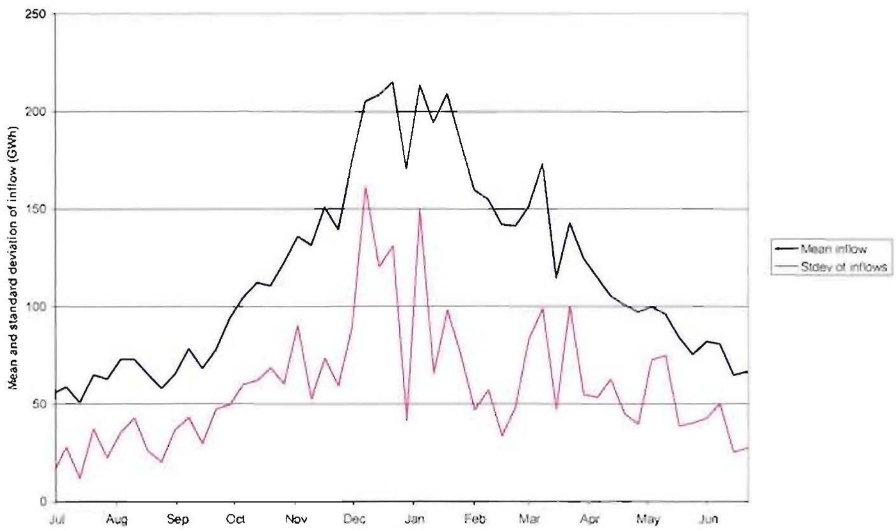


Figure 8.2: Inflow data

8.2 The risk neutral base case

A standard SDP approach was used to solve this version of the reservoir management problem with the firm assumed to be risk neutral in wealth. A value of storage function, $V(s^{T+1})$, was used to reflect the monetary value of water at the end of the horizon. An equilibrium $V(s^{T+1})$ was determined by repeatedly solving the optimisation and using $V(s^{T+1})$ from each iteration as $V(s^{T+1})$ for the next. The equilibrium $V(s^{T+1})$ was found when the mean squared deviation between the successive functions was less than 0.1. No more than 10 iterations were required, and this process was performed for each contract level.

Figure 8.3 and Figure 8.4 show the CDFs of end-of-horizon wealth and end-of-horizon storage from the RN simulations for each contract level (C).

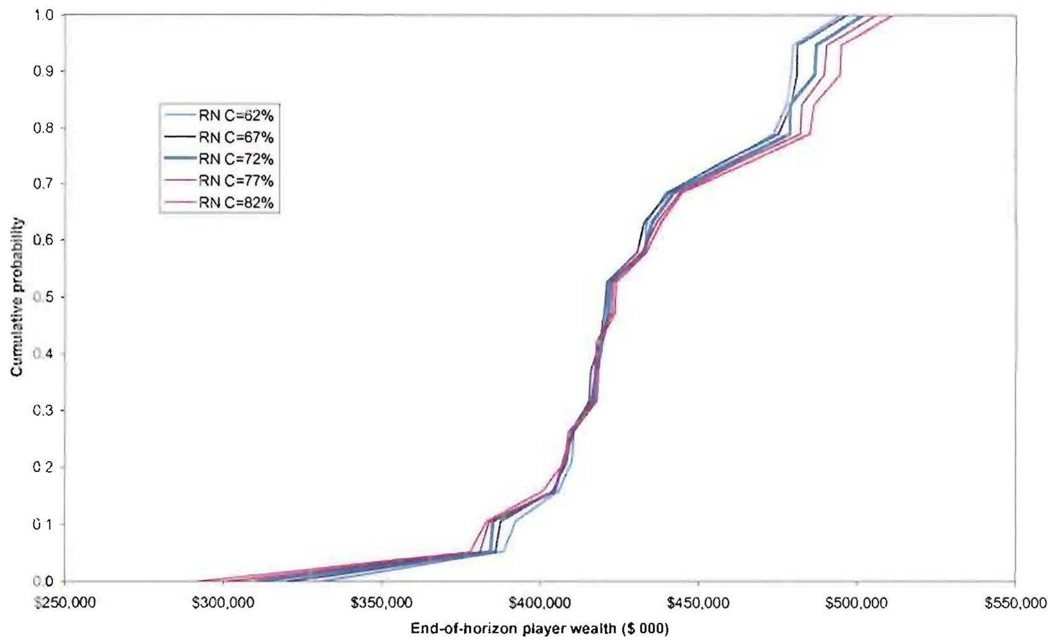


Figure 8.3: CDF of RN EOH wealth with contract level varied

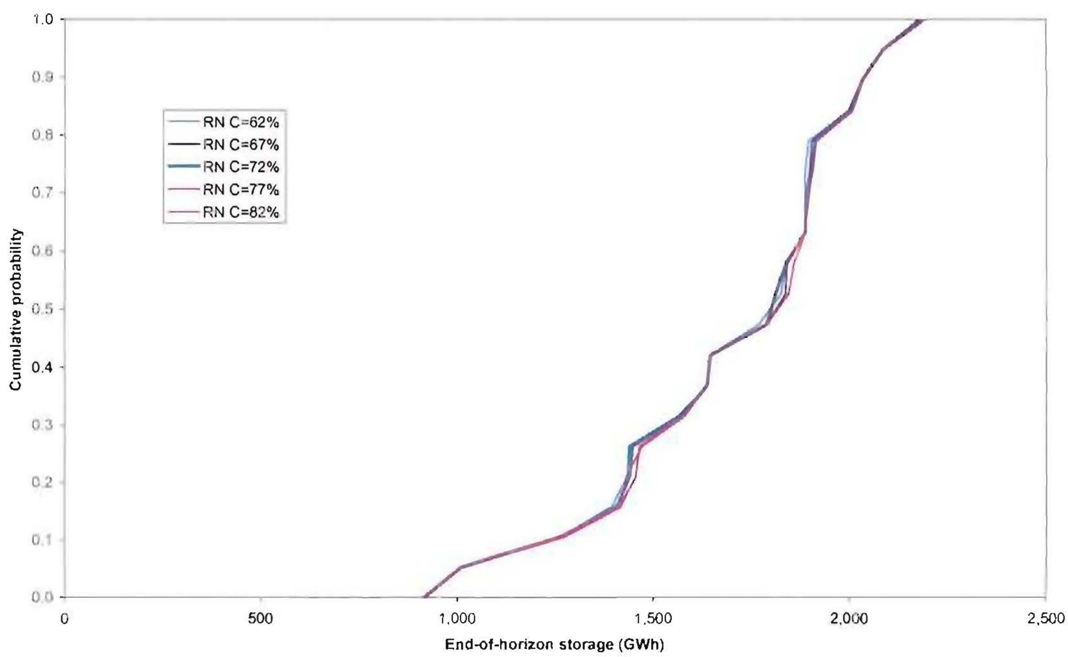


Figure 8.4: CDF of RN EOH storage with contract level varied

The base case for these experiments is considered to be where the firm is 72% contracted. A strike price of $48/MWh was used in all periods. This price was determined by averaging all the spot prices from the 20 simulated cases when the firm was risk neutral and contracted for 72% of its release capacity, or effectively 165% of its average output.

8.3 Utility functions

The definition of the end-of-horizon value surface is the same as was used in i.e.,

$$f_{T+1}(w_{T+1}, s_{T+1}) = U_W(w_{T+1}) + U_S(V(s_{T+1})) \tag{8.1}$$

where

U_W is a function reflecting the firm's utility derived from the level of reservoir wealth at the end of the horizon.

U_S is a function reflecting the firm's utility derived from the level of storage at the end of the horizon.

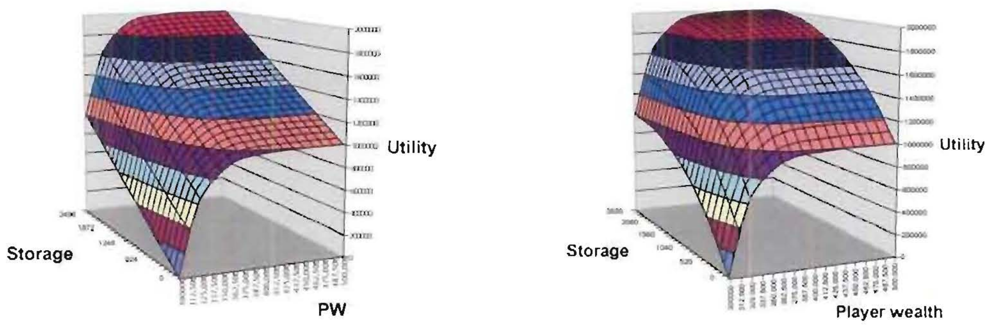
The utility of reservoir wealth is handled by defining a utility curve over a suitable range of FW values, and extrapolating for FW values outside this range. Experiments were performed for the following combinations of contract level, utility of wealth, and utility of storage. Summary statistics for all experiments are detailed in Appendix 3.

- The firm's contract level: 62%, 67%, 72%, 77%, 82%[5].
- The utility of wealth function – W0, W1, W2, W3, and W4 - where all the utility curves are non-decreasing with non-increasing marginal utility. W0 has the least curvature (linear) and W4 the most curvature.
- The utility of storage function – S0, S1, S2, S3, and S4 - where all the utility curves are non-decreasing with non-increasing marginal utility. S0 has the least curvature (linear) and S4 the most curvature.

Three combinations of U_W and U_S receive the most attention in the following discussion of simulation results, the reason being that they reflect three extreme scenarios. The first is termed the risk neutral (RN) case, where the firm is risk neutral

[5] An firm contracted for more than its expected output has incentives to keep the spot price as low as possible so as to avoid large difference payments which can occur if relatively expensive thermal stations are marginal. This over-contracted scenario could reflect a situation, for example, where the hydro firm is a utility is comprised of generation and retail businesses, with retail load exceeding expected generation.

with respect to wealth and $V\left(s^{T+1}\right)$ reflects the value of water at the end of the horizon (this is the base case described earlier and does not use a utility function). The other two combinations, S0W4 and S2W4, are illustrated in Figure 8.5.



S0W4 **S2W4**

Figure 8.5: Combinations of U_W and U_S

8.4 Impact of risk aversion on firm/system performance

These results all assume a contract level of 72% and use the same contract volumes and strike prices as defined for the RN base case in Section 8.2. It is important to note that this contract level results in the firm being over-contracted, so the firm has the incentive to keep prices down. Recall also that the assumption made here is that the hydro firm has complete control over the spot price (assuming all release alternatives are feasible) because it knows the reaction of all other firms in the market.

Before discussing the results, recall that utility is defined over both ending wealth and ending storage by an asymmetric function. We should therefore expect to see the ending storage and wealth outcomes reflect the relative utility as defined by the utility functions. Because the storage and wealth depend on each other, any gains in one dimension will be expected to be obtained at the other's expense, as was shown and discussed in . Decreasing the variability of the ending wealth values results in an increase in the variability of ending storage values, and vice versa. The same logic applies for the mean. Because the utility functions are asymmetric (e.g. Figure 8.5), the nature of the impacts on the distributions of end-of-horizon wealth and storage should not be expected to be symmetric either. For example, in the case where the firm has more risk aversion towards end of horizon wealth values relative to end-of-horizon storage values (e.g., the S0W4 utility function), we would expect that ending wealth values would be lower and have less variability compared to a case where the opposite is true (e.g., the S4W0 utility function).

Firstly, we consider the impacts of the utility functions on end of horizon values for the base case 72% contract level. Figure 8.6 plots average annual wealth against expected ending storage (LHS y-axis) and the standard deviation of the expected annual wealth (RHS y-axis) from the simulation results corresponding to each combination of wealth/storage utility function (W0S0, W0S1, ... W4S3, W4S4).

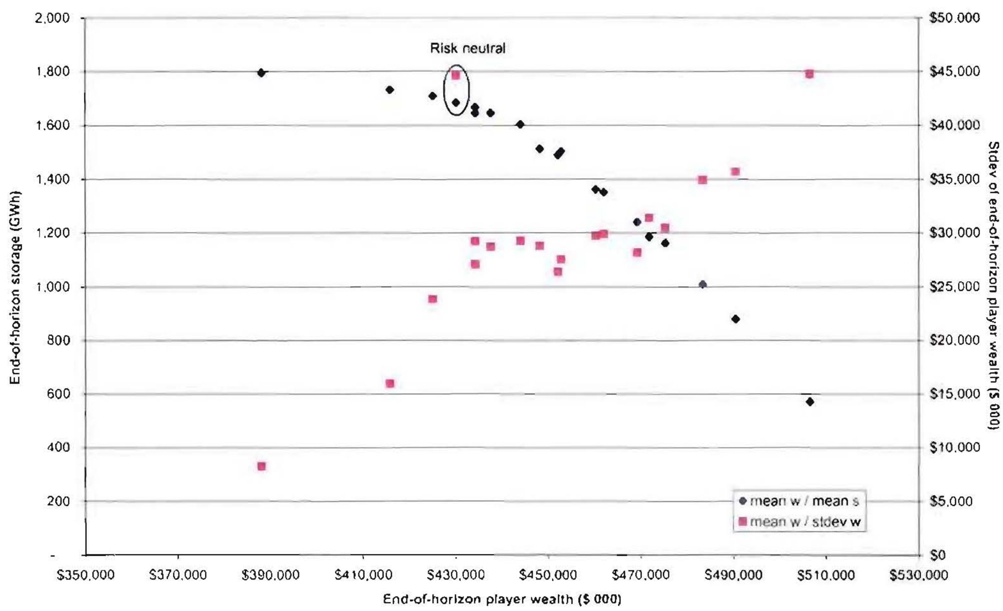


Figure 8.6: Wealth/storage trade-off (C=72%)

The form of the trade-off between these end-of-horizon outcomes is fairly straightforward and intuitive:

- a higher expected end-of-horizon wealth is highly variable and corresponds to a lower expected end-of-horizon storage.
- a lower expected end-of-horizon wealth can be achieved with relative certainty, and corresponds to a higher expected end-of-horizon storage.

These results have a pattern consistent with those illustrated and discussed in (see for example Figure 6.15).

We consider now results obtained using the S0W4 and S2W4 utility functions as well as that of the RN base case. Figure 8.7 and Figure 8.8 show the CDFs for end of horizon wealth and storage.

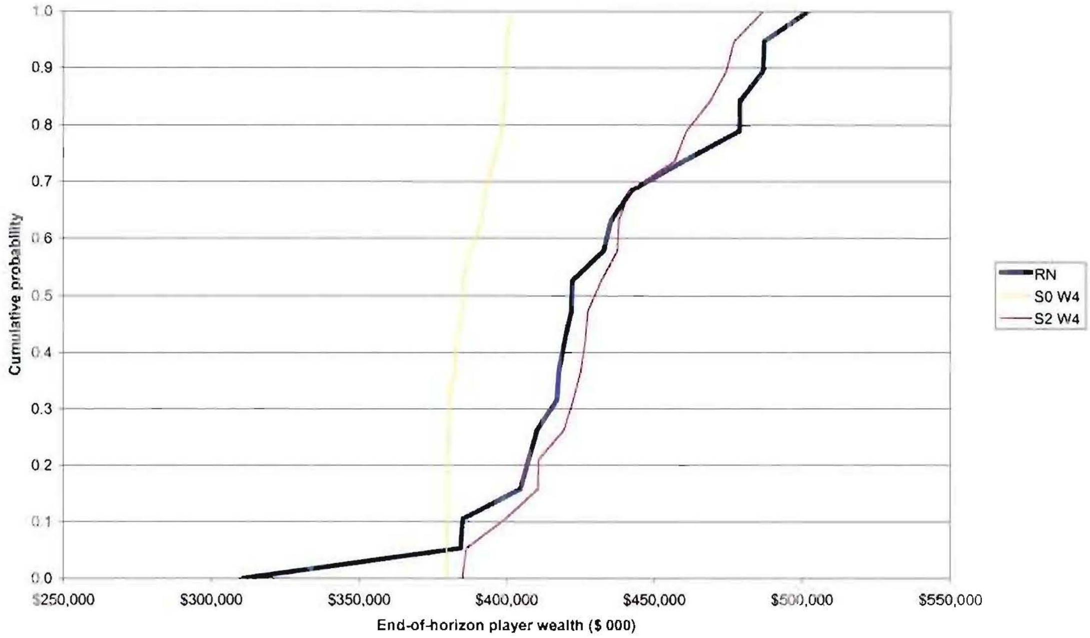


Figure 8.7: End-of-horizon wealth CDFs (C=72%)

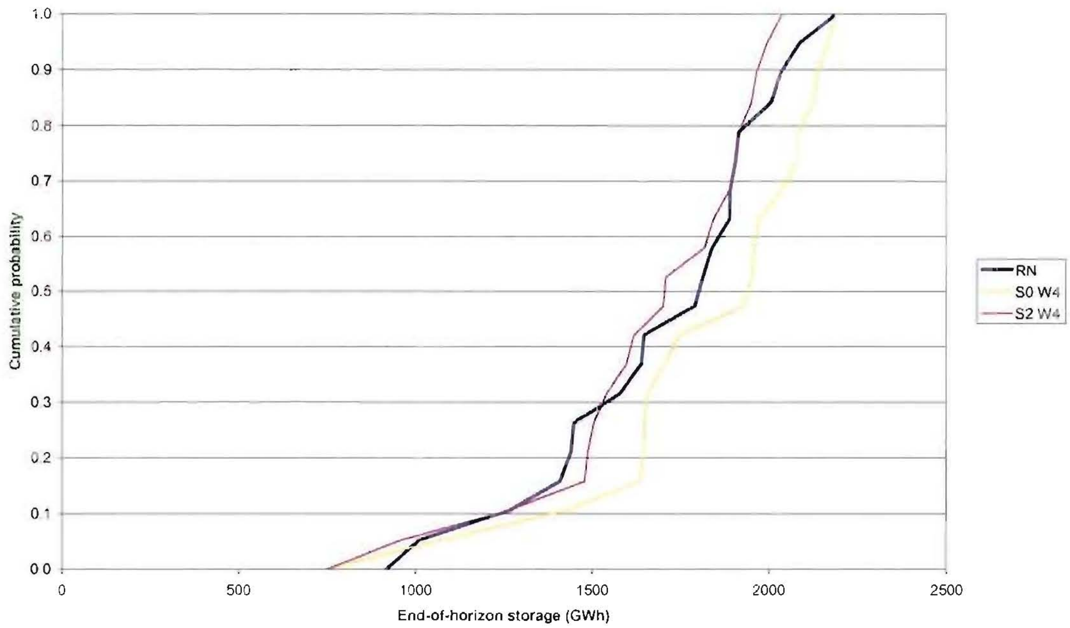


Figure 8.8: End-of-horizon storage CDFs (C=72%)

The impact of risk aversion on the wealth CDFs is that they compress and become more upright, with minimum wealth increasing, maximum wealth decreasing, and mean wealth increasing by \$4m using S2W4 and decreasing by \$42m for S0W4. The standard deviations of end-of-horizon wealth for the three cases are \$45m (RN), \$28m (S2W4), and \$8m (S0W4). See Appendix 3 for more detailed results for those cases. It is clear from the end-of-horizon wealth CDF that the lowest RN outcome ($310m) is far lower than those of the other cases, and hence is a primary cause for the large standard deviation of end-of-horizon wealth for the RN case[6]; the 5th percentile figures for the S0W4 and RN cases are close in comparison (see Table 8.2, which follows). As in , and as reflected in Figure 8.6, these end-of-horizon wealth distributions must be considered in conjunction with the distributions of end-of-horizon storage, since, as has been discussed, the two are linked.

Figure 8.9 and Figure 8.10 plot a risk averse CDF against the RN CDF including the 5% (dashed), 25% (dotted), 50% (solid), 75% (dotted), and 95% quartiles.

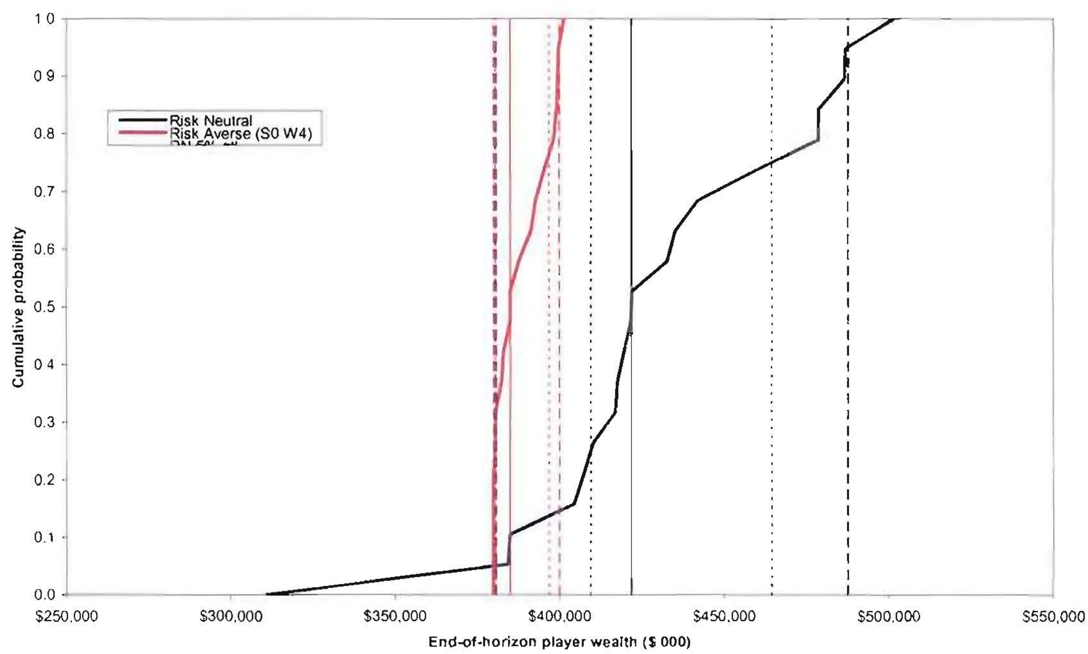


Figure 8.9: RN and S0W4 end-of-horizon wealth CDFs (C=72%)

[6] See the circled points in Figure 8.6.

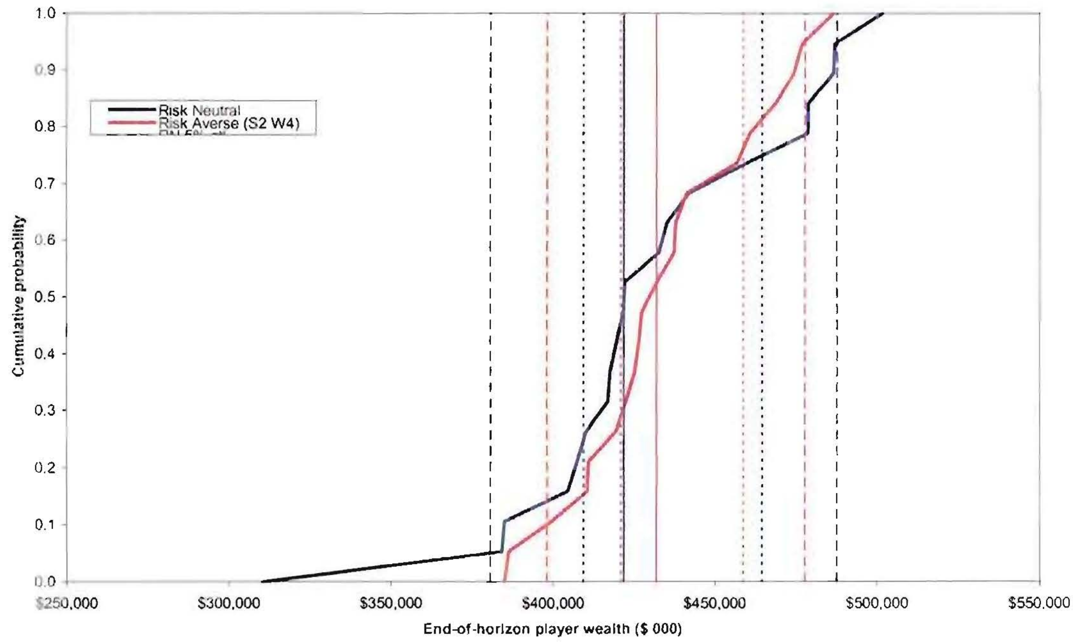


Figure 8.10: RN and S2W4 end-of-horizon wealth CDFs (C=72%)

	RN	S0 W4	*Diff from and % of RN*
PW: Minimum	$ 310,807	$ 379,811	$69004 22%
PW: 5% percentile	$ 380,610	$ 379,866	$-744 0%
PW: 25% percentile	$ 409,460	$ 380,429	$-29031 -7%
PW: Median	$ 421,823	$ 384,993	$-36829 -8%
PW: 75% percentile	$ 464,510	$ 396,768	$-67742 -14%
PW: 95% percentile	$ 487,579	$ 399,896	$-87682 -17%
PW: Maximum	$ 501,539	$ 401,375	$-100163 -19%
PW: Mean	$ 430,072	$ 387,999	$-42072 -9%
PW: Standard deviation	$ 44,600	$ 8,227	$-36372 -81%

Table 8.2: End-of-horizon wealth results (C=72%)

The storage CDFs have similar shapes, and this is reflected by the mean storage trajectories (Figure 8.11) and mean release trajectories (Figure 8.12) having similar profiles. The starting storage level for each simulation is 2080GWh, and corresponds to the first week of July. The storage profiles are as we would expect, falling over winter/spring (demand higher), rising over spring/summer/autumn (demand lower), then falling again in autumn/winter (demand higher).

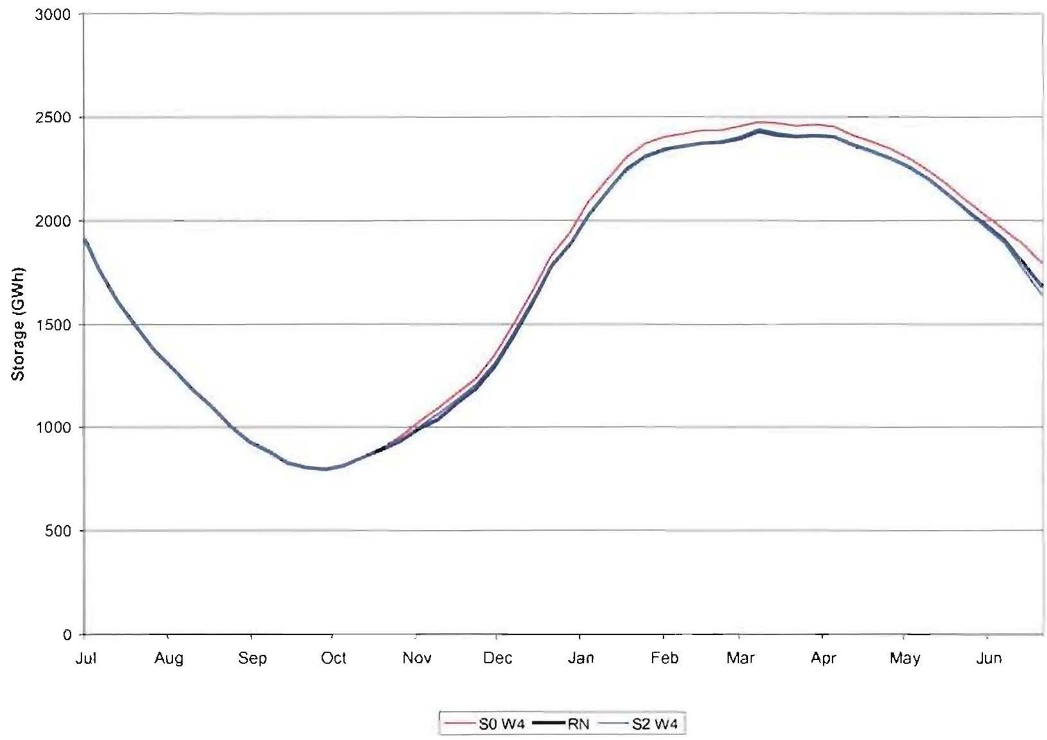


Figure 8.11: Mean storage trajectories

It was shown earlier that compared to the simulation results for the RN case, those for the S0W4 case had a lower mean and standard deviation of end-of-horizon wealth. It was also commented that since utility is defined over end-of-horizon wealth and storage, changes in the distribution of wealth (storage) is inversely related to changes in storage (wealth). This relationship is reflected by the average storage trajectories in Figure 8.11, with the end-of-horizon wealth outcomes appearing to be achieved by holding more water in storage, on average, to reduce the chance of low inflows which might subsequently result in high difference payments due to relatively expensive thermal stations being required to satisfy demand. This policy produces in a higher ending EOH storage (1,795GWh) compared to the RN case (1684GWh) and the S2W4 case (1645GWh). For S2W4, mean end-of-horizon wealth actually increased compard to the RN case, but this is matched with a decrease in mean end-of-horizon storage.

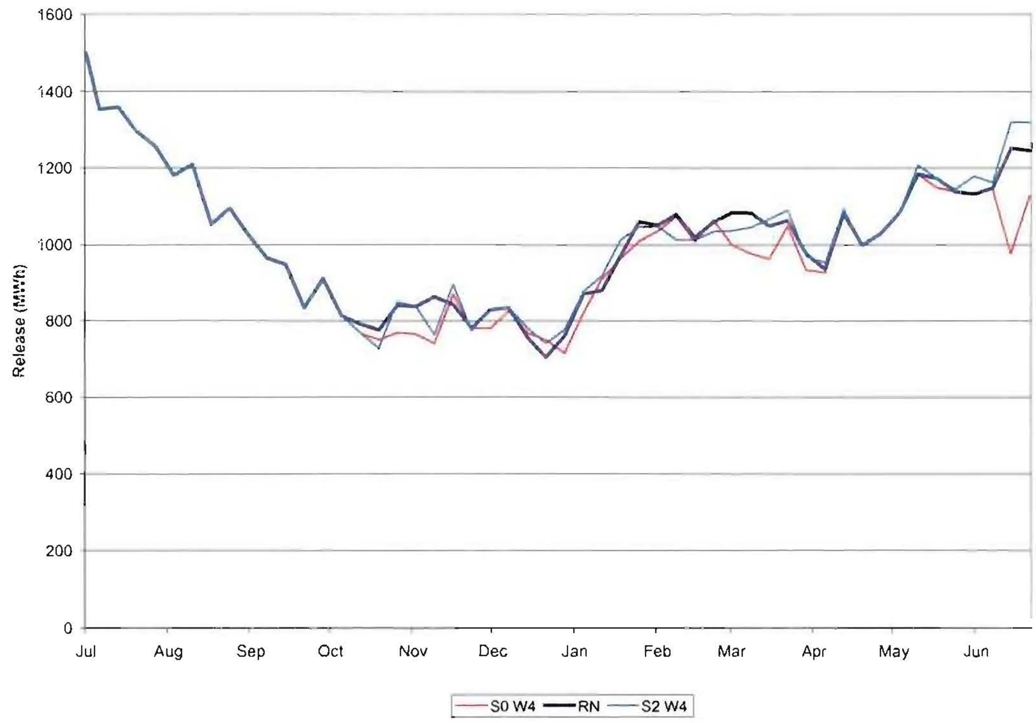


Figure 8.12: Mean release trajectories

The contract quantity in each week is 1688MWh, so the release schedules in Figure 8.12 reflect the situation (discussed earlier) that the firm is over contracted and always buying back generation from the market. Thus, it will have the incentive to push the spot price as low as possible, and ensure that high spot price events occur rarely, since these will result in large difference payments and will therefore lower wealth. The mean spot price (Figure 8.13) is usually less than \$50, indicating that the firm is releasing enough to ensure that Huntly units 2 and 3 (\$266/MWh) remain out of the dispatch, on average, and hence that the most expensive thermal station is New Plymouth (\$49/MWh). The large difference between these two marginal costs means that release will be highly linked to the demand profile. There is a step in the supply curve of approximately 500MW at a marginal cost of approximately \$49, so it is the optimal release for a large proportion of 'moderate' storage/wealth combinations, as simulated here.

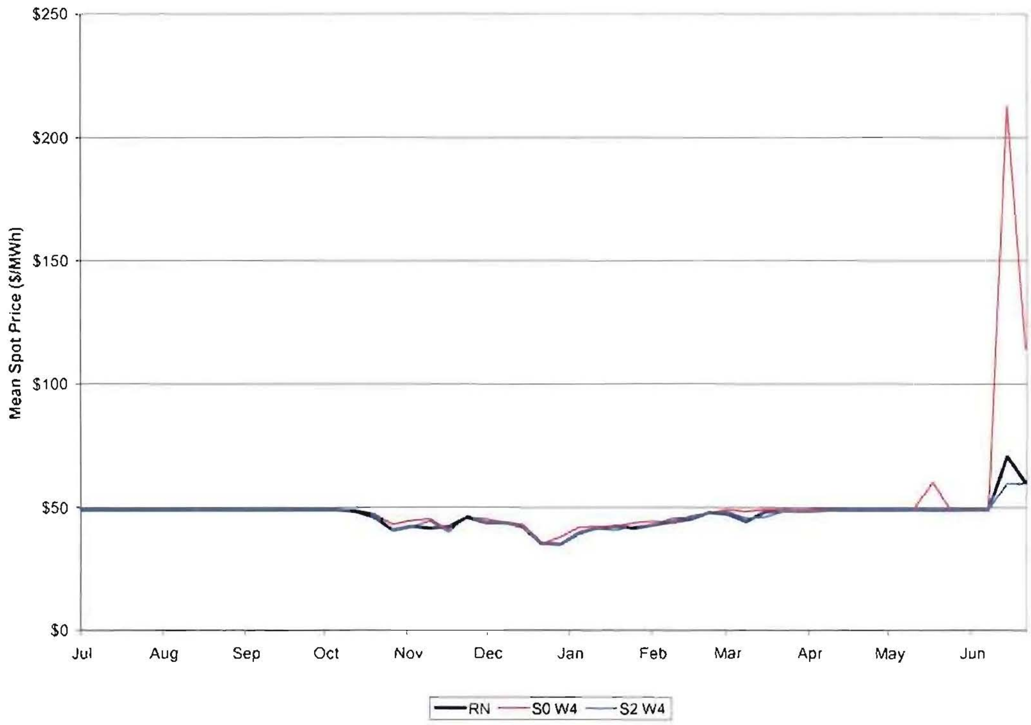


Figure 8.13: Mean spot price

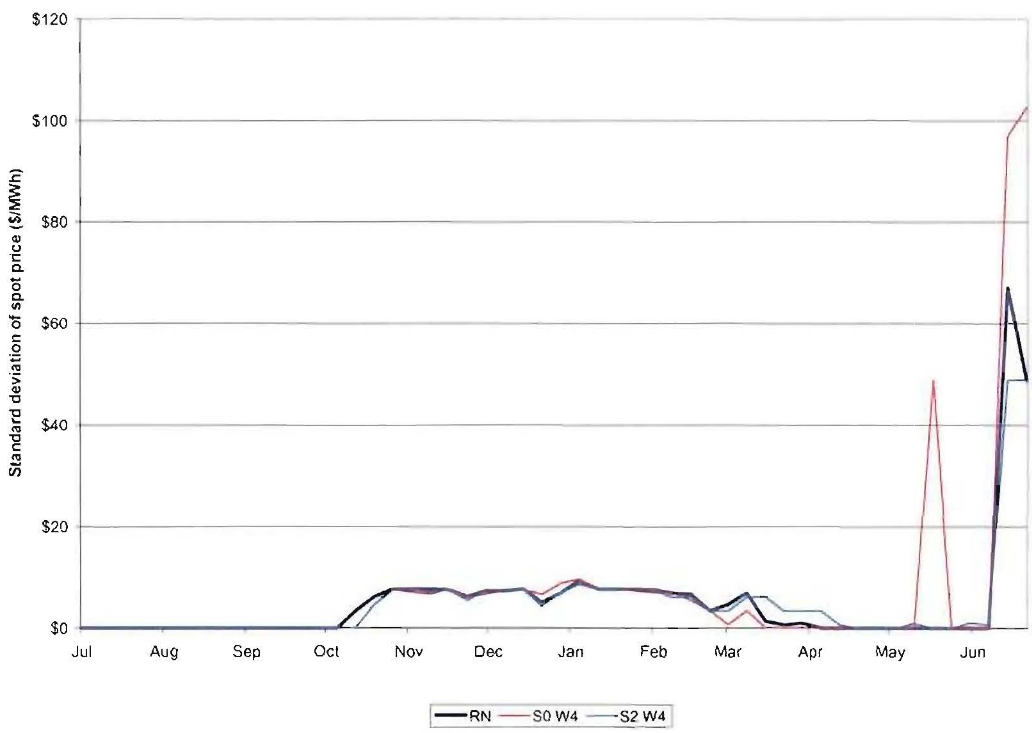


Figure 8.14: Standard deviation of spot price

8.5 The impact of risk aversion on other firms

When the firm is risk neutral, there is relatively little variability in competitor mean generation levels (Figure 8.15), most of the variability occurring in summer when inflows are high and demand is low (Figure 8.16). Thus the effect of SOW4 utility function relative to the RN case is minimal.

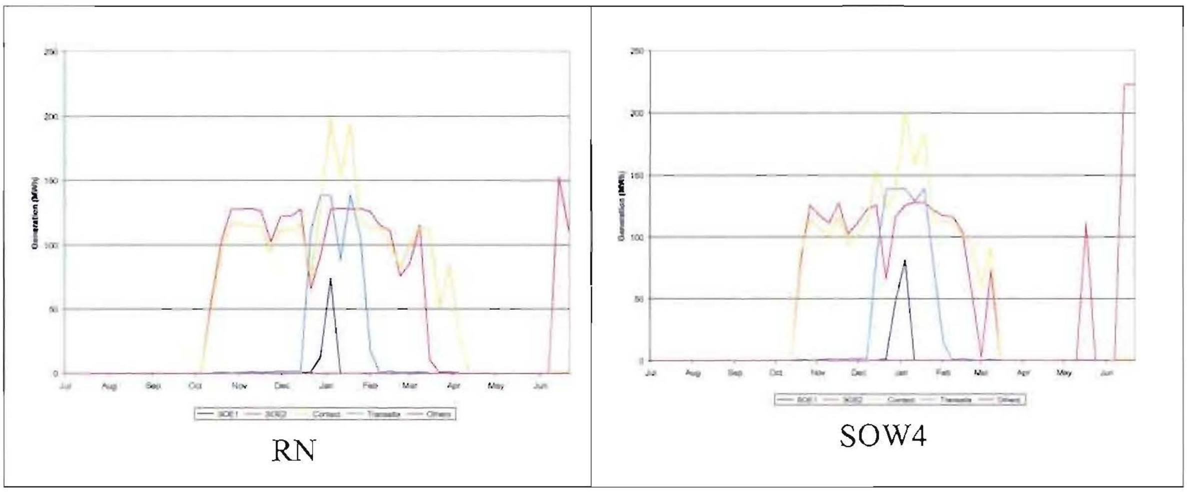


Figure 8.15: Mean competitor generation (C=72%)

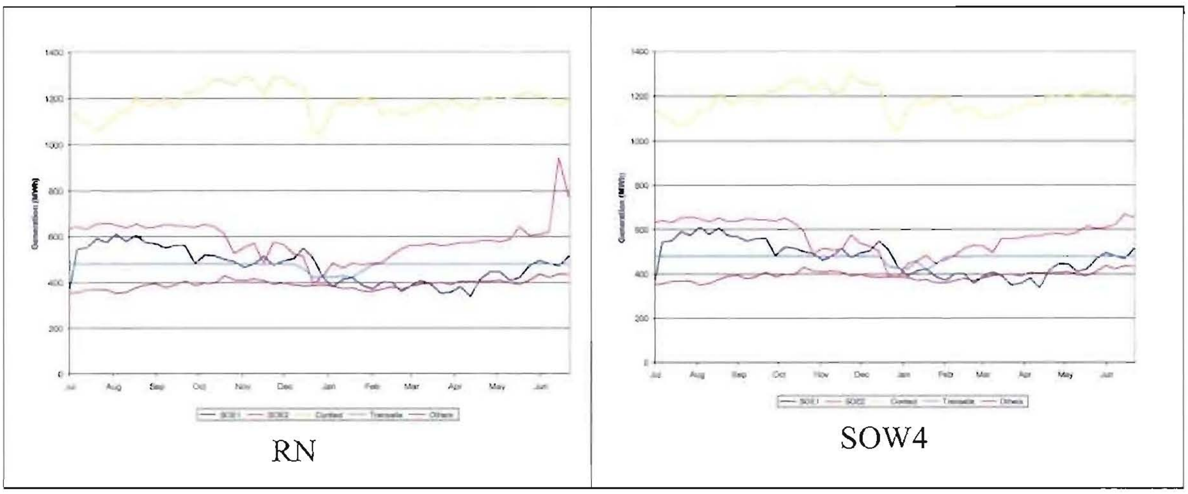


Figure 8.16: Standard deviation of competitor generation (C=72%)

Table 8.3 shows average annual generation and Table 8.4 shows average annual profit for each firm. Generation is in units of GWh while profits are in units of ($ 000). The spot price is in units of $/MWh.

		C=62%				C=72%				C=82%			
		RN		S0W4		RN		S0W4		RN		S0W4	
		Mean	St dev	Mean	St dev	Mean	St dev	Mean	St dev	Mean	St dev	Mean	St dev
Competitors	SOE1	4,125	2	4,123	10	4,122	15	4,119	21	4,123	9	4,119	19
	SOE2	5,015	254	5,191	133	5,001	260	5,145	146	5,002	263	5,125	155
	Contact	10,300	279	10,391	260	10,325	267	10,394	257	10,339	260	10,411	254
	Transalta	4,141	112	4,136	126	4,147	100	4,138	123	4,149	99	3,974	113
	Others	3,438	-	3,438	-	3,438	-	3,438	-	3,438	-	3,438	-
	Shortage	-	-	-	-	-	-	-	-	-	-	-	-
SOE3	Generation	8,954	638	8,694	484	8,940	602	8,738	505	8,921	586	8,738	499
	Storage	1,696	192	1,739	203	1,700	193	1,736	201	1,703	194	1,739	202
Market	Price	$51.35	$3.03	$58.29	$2.23	$45.99	$3.14	$51.52	$2.16	$46.94	$3.19	$50.04	$2.12

Table 8.3: Average annual generation with firm utility and contract level varied

		C=62%				C=72%				C=82%			
		RN		S0W4		RN		S0W4		RN		S0W4	
		Mean	St dev	Mean	St dev	Mean	St dev	Mean	St dev	Mean	St dev	Mean	St dev
Competitors	SOE1	$105,270	$12,564	$24,476	$62	$90,995	$12,944	$24,451	$124	$90,837	$13,116	$24,452	$115
	SOE2	$127,293	$14,300	$30,814	$792	$103,653	$14,562	$30,543	$869	$103,544	$14,686	$30,422	$922
	Contact	$304,803	$30,149	$61,681	$1,543	$261,434	$31,028	$61,699	$1,528	$261,099	$31,408	$61,798	$1,505
	Transalta	$89,638	$12,550	$24,549	$746	$71,494	$12,926	$24,563	$730	$71,368	$13,061	$24,587	$668
	Others	$139,070	$11,039	$20,406	$0	$125,540	$11,394	$20,406	$0	$125,382	$11,560	$20,406	$0
	Shortage	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
SOE3	Wealth	$430,178	$38,960	$382,089	$2,965	$430,072	$44,600	$387,999	$8,227	$431,294	$50,779	$393,853	$11,861

Table 8.4: Average annual profits with firm utility and contract level varied

The results from using the S0W4 function result show a slight decrease in the firm's generation relative to the RN results, and this has the largest impact on the annual generation levels of SOE2 (increased mean and reduced standard deviation) and to a lesser extent Contact and Transalta (decreased mean and increased standard deviation). SOE2 is most affected by the change in contract and risk aversion, and this is because it owns Huntly unit 4 ($46.20/MWh) and Huntly units 2&3 ($266.90) which will be marginal stations in the dispatch.

Another consequence of risk aversion is that with the firm generating less (so as to store water as an insurance against high cost events), more expensive stations must be called on now and again to meet demand, resulting in higher spot prices. From Figure 8.13, though, it is apparent that this effect resulted in a minimal change in spot prices. The reason is that the hydro firm is over-contracted so has an incentive to minimise the difference payments, and hence the extent to which prices rise above the strike price.

8.6 The impact of contracts and risk aversion

The impact of the three 62%, 72%, and 82% contract levels on RN and SOW4 end-of-horizon wealth is shown in Figure 8.17, with Figure 8.18 showing the corresponding effect on storage. At C=62%, the firm faces less exposure to the spot market (relative to the 72% and 82% cases), and hence to situations which will increases the variability of end-of-horizon wealth outcomes. Decreasing the contract level results in the CDFs compressing and become more vertical. For S0W4, the wealth CDFs shift to the left slightly, but this is compensated for by a shift to the right in the storage CDF.

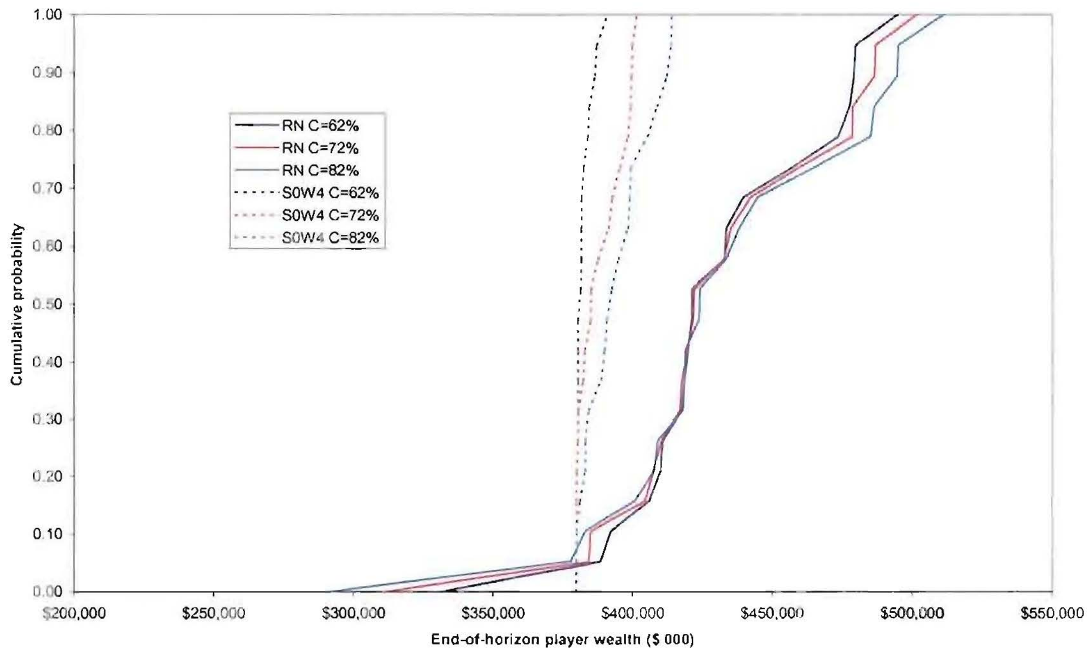


Figure 8.17: RN and S2W0 wealth CDFs with contract level varied

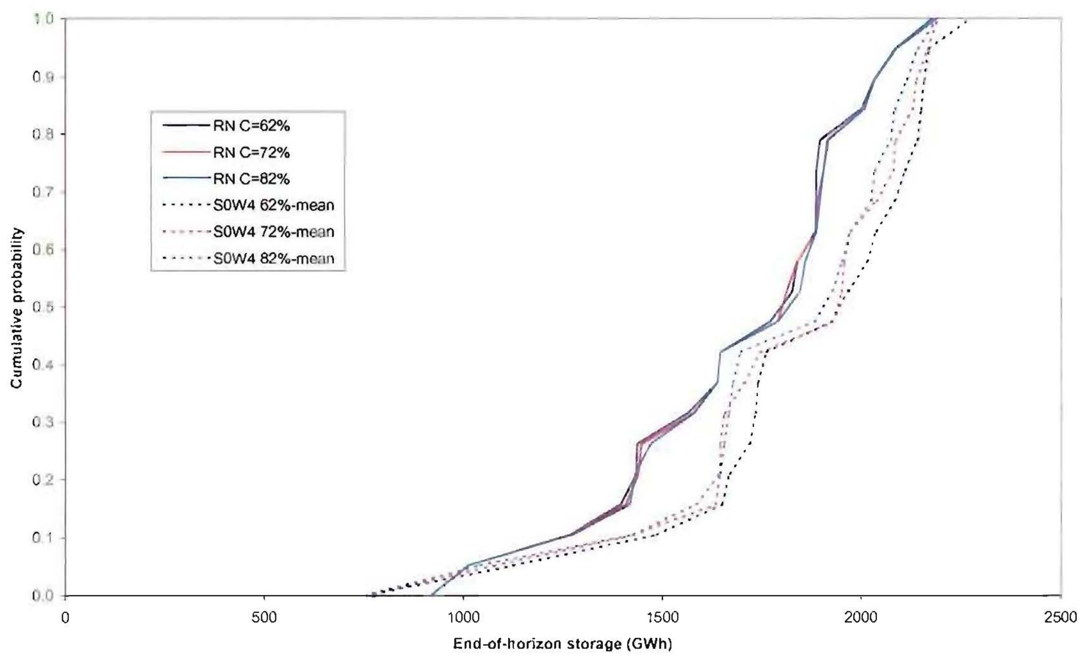


Figure 8.18: RN and S2W0 storage CDFs with contract level varied

The combined impact of risk aversion and contracting is best illustrated by comparing the two most extreme scenarios: S0W4&C=62% and RN&C=82%. The mean wealth values are \$382m and \$431m, while the standard deviations are \$3m and \$50.8m, respectively. The mean spot prices are \$58.29 compared to \$46.94, while annual generation is largely unchanged (8,694GWh compared to 8,921GWh). As has been mentioned earlier, this reduction in end-of-horizon wealth (but with less variability) is compensated for by a shift in the mean end-of-horizon storage CDFs (increased mean) as well as an increase in its variability.

Regardless of the contract level, simulation results produced using the S0W4 risk aversion function produce results which significantly decrease the mean wealth and standard deviation compared to the risk neutral (RN) cases. The range of contract levels used here had a relatively minor impact on the distributions of ending wealth compared to the impact of the utility functions.

8.7 Summary

In this chapter, the impacts of risk aversion were investigated for a reservoir operated as a price setter in a wholesale electricity market. Concave utility functions had the expected impact on wealth, reducing the variation of end-of-horizon wealth outcomes, in some cases quite dramatically. This was at the expense of increasing the variability of end-of-horizon storage levels, though the increase was small compared to the impact on wealth. The change in mean wealth and storage was influenced by the nature of the trade-off between storage and wealth in the utility function. For a fixed contract level, increasing the concavity of U_W, and holding U_S constant (or, increasing the relative risk aversion towards end-of-horizon wealth) produces the following results:

- μ_{W_T} and σ_{W_T} decrease.
- μ_{S_T} and σ_{S_T} increase.
- The range of end-of-horizon wealth outcomes decreases. Both the maximum (best) and minimum (worst) EOH-W tend towards the mean, though the increase in the minimum EOH-WW value is larger than the decrease in the maximum EOH-W.

- Storage trajectories are higher on average and releases lower on average. Holding storage is a mechanism for hedging against the impacts of future inflow uncertainty on revenue given the contract level.

- Spot prices tend to increase due to lower release levels.

As contract levels decrease and tend toward the expected output:

- Average release decreases marginally and average spot prices increase, reflecting the decrease in the incentives to keep spot prices low.

- μ_{W_T} decreases marginally, presumably due to the over-contracting situation resulting in a balancing of contract revenues and difference payments (both positive and negative) for different over-contracted levels.

- σ_{W_T} decreases, presumably because the impacts of low inflows (and the subsequent low release and high difference payments) on wealth are reduced.

- The impact of risk aversion on releases lessens because the weekly benefit function becomes more 'jagged' and plays a more dominant role in the trade-off between wealth and storage implied by the cost-to-go function.

For the scenarios considered here, altering the risk aversion functions had more influence on the distributions of end-of-horizon wealth compared to altering the contract levels (and holding the risk aversion functions constant). As was shown and discussed, the variability of the end-of-horizon wealth did decrease slightly as the contract level was decreased, reflecting the ability of the hydro firm to reduce the frequency of situations which resulted in large difference payments, and hence reduce the variability of end-of-horizon wealth outcomes.

Competitors were dispatched in order of marginal (offer) cost in order to meet demand, thus their generation and profit levels were directly related to the hydro firm's release levels. Firms owning stations with marginal costs around the average marginal cost were most affected because slight changes in the hydro firm's release levels resulted in them being removed, or included, in the dispatch. Firm SOE2 was most affected by changes in the hydro firm's risk aversion and contract levels due to its ownership of the Huntly4 unit, which was often marginal.

Overall, using a risk averse utility curve resulted in release schedules which substantially reduce the variability in end-of-horizon wealth outcomes while not greatly affecting the distributions of end-of-horizon storage. Although the firm possesses a strong influence on the spot price (due to all other firms being treated as a fringe), the results do provide a basis for comparison with models that use a more realistic form of market interaction, and suggest the merit of more research in this area, and in particular, for scenarios where the nature of competition, and the contract levels, are more realistic.

Chapter 9

Extensions

9.1 Introduction

The previous chapters have described and illustrated SUMDP for a reservoir firm operating in 'regulated' and 'deregulated' environments. In the latter case, the reservoir firm was a dominant firm and the other firms were assumed to be price taking. In reality, though, these firms may compete and have an influence over the market price. In Section 9.2, a Stackelberg leader/follower model is developed where fringe firms compete for residual demand rather than being price takers. The previous cases have also assumed that the reservoir firm only produces hydro generation. Section 9.3 discusses the case where the dominant firm owns thermal stations in addition to its hydro capacity.

9.2 A Stackelberg leader/follower model

Scott (1997) presents a Cournot analysis for the case where profit maximising (and implicitly risk neutral) contracted firms compete for market share by adjusting their quantities. A Nash equilibrium is found at the generation levels at which neither firm has any incentive to adjust their quantity offered given the offer of the other firms, which have also determined their offers using the same process. Scott used this model to represent weekly operating behaviour and embedded it in a Dual Dynamic Programming model for reservoir management. One of the firms generates power from hydro generation and faces weekly inflow uncertainty. Instead of using a storage state variable in each period, Scott uses the marginal water value (MWV) associated with different storage levels. The MWV reflects the value of storing water and using it in a later period rather than releasing it now. Thus, when storage is high the MWV is low, and when storage is low the MWV is high. This is relevant because the MWV is treated as the hydro firm's marginal cost, so the market equilibrium and associated firm generation levels can therefore be determined using the Cournot equilibrium equations where the firms compete on the basis of quantity and not price.

A variation from Scott's approach is to use a benefit function based upon a follower/leader game where the hydro firm is the leader and the other firms are followers. The leader offers some quantity to the market. This has the effect of reducing demand by the offer quantity, which is known by all firms. The followers then compete in a Cournot fashion for the remaining demand until a (Nash) equilibrium is found. An output from this equilibrium is the market price, which is used to derive the hydro firm's (weekly) profit function. This profit function can then be used to calculate end-of-period wealth, which is used to determine the optimal release.

The main assumption of this approach is that any level of hydro release is accepted in the market dispatch, which is reasonable if the hydro manager adopts a strategy of offering generation at a price of zero, or has some other mechanism for making this possible. Although the follower firms determine the price via their competition for 'follower' demand, the form of their competition is known by the hydro firm. Therefore, the hydro firm knows the market price, and the corresponding profit, that will result from any given level of hydro release.

Scott also presented the Cournot equilibrium conditions for constant elasticity demand which assume a constant marginal cost and that each firms' generation is unbounded. In reality, a firm's generation is bounded and its marginal costs may not be constant. Examples of non-constant marginal costs include linear marginal cost curves and stepped supply curves.

Scott presents the equilibrium equations for the case where a firm's marginal cost is fixed (corresponding to a horizontal segment of its supply curve) and when its generation is fixed (corresponding to a vertical segment of its supply curve). Scott's technique for determining what he termed "admissible solutions" was to find solutions for different combinations of assumptions regarding each firm's supply curve, and then test whether solutions were consistent with the marginal costs used to derive them. See Chapter 3 of Scott (1997) for further discussion of this[7].

9.2.1 Linear demand

Using the same notation as in previous chapters, the market levels (m subscript) of marginal cost, generation and contracts used in Scott's Cournot equilibrium equations are:

$$\bar{c}_m = \sum_{i=1}^{I} \frac{c_i}{I},\ g_m = \sum_{i=1}^{I} g_i,\ f_m = \sum_{i=1}^{I} f_i \tag{9.1}$$

where c_i, g_i, and f_i correspond to firm i's marginal cost, generation level and contract level (respectively). This analysis is assumed to be for a particular period, so the t subscript is dropped unless otherwise necessary.

Now consider the case where demand is assumed to be linear and represented by an inverse linear demand function of the form:

$$p(g_m) = p_0 + \rho(g_m - g_0) \tag{9.2}$$

where (p_0, g_0) is the reference point from which the price can be thought to deviate depending on the difference between g_m and g_0. The Cournot equilibrium market price and generation level are:

$$p^* = \frac{p_0 + \bar{c}_m I + \rho f_m - \rho g_0}{I+1} \tag{9.3}$$

$$g^* = \frac{\bar{c}_m I - p_0 I + \rho g_0 I + \rho f_m}{\rho[I+1]} \tag{9.4}$$

The individual firm generation levels are:

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[7] In fact, the same problem can be represented as a Mixed Complementarity Problem (MCP) and solved quickly

$$g_i^* = f_i + \frac{c_i[I+1] - p_0 - \bar{c}_m I - \rho f_m + \rho g_0}{\rho[I+1]} \tag{9.5}$$

Given the linear demand curve described earlier, a 'follower' demand curve, $p_F(g_F, q_k^t)$, can be created by subtracting q_k^t, the 'leader' generation level, from the market demand curve i.e.,

$$p_F(g_F, q_k^t) = p_0 + \rho(g_F - (g_0 - q_k^t)) \tag{9.6}$$

where g_F is total follower generation and q_k^t is a discrete value of q^t, with $k \in K$ and K being the set of feasible discretised hydro release levels.

The 'follower' Cournot equilibrium price is determined as before with the g_0 terms replaced by $(g_0 - q_k^t)$. The 'follower' market consists of $I'=I$-1 firms and the 'follower' market contract level, f_m', excludes the hydro firm's contract level. Thus we have:

$$p_F^* = \frac{p_0 + \bar{c}_m I' + \rho f_m' - \rho(g_0 - q_k^t)}{I'+1} \tag{9.7}$$

$$g_F^* = \frac{\bar{c}_m I' - p_0 I' + \rho(g_0 - q_k^t)I' + \rho f_m'}{\rho[I'+1]} \tag{9.8}$$

and equilibrium generation levels for each firm:

$$g_i^* = f_i + \frac{c_i[I'+1] - p_0 - \bar{c}_m I' - \rho f_m' + \rho(g_0 - q_k^t)}{\rho[I'+1]} \tag{9.9}$$

Given the equilibrium follower generation, the market generation will be $g^* = g_F^* + q_k^t$. The market price corresponding to g^* is therefore:

$$p(g^*) = p_0 + \rho(g_F^* + q_k^t - g_0) \tag{9.10}$$

The inverse market demand curve is just the follower inverse demand curve shifted to the right by q_k^t, so the equilibrium price remains the same i.e,

$$p(g_F^* + q_k^t) = p_F(g_F^*, q_k^t) \tag{9.11}$$

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using commercial non-linear solvers.

as shown in Figure 9.1.

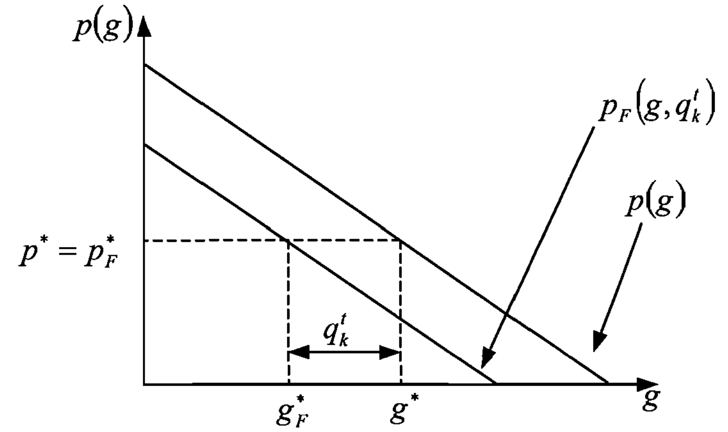


Figure 9.1: Equilibrium market and follower price and generation

The form of the solution is similar to that discussed by Scott, who discusses it in the context of checking for an admissible solution if a firm is operating on a vertical section of its supply curve. In this case, though, it is the hydro firm which has its output fixed, and this output is feasible by definition. If the followers have stepped supply curves, it will be necessary to find an admissible follower solution for each q_k. Scott (1997) discusses the uniqueness of these solutions for linear and constant elasticity demand.

For the follower/leader game of interest here, the above analysis can be performed for the range of discrete q_k^t. Assuming admissible follower solutions are found, there will be a price (p^*) associated with each release. In effect, this analysis would result in a 'leader' demand curve, $p_L(q^t)$, which reflects the market price for different hydro ('leader') generation levels.

If the competing firms each have a single marginal cost and unbounded generation, then $p_L(q^t)$ can be derived analytically as follows. The slope of the leader demand curve, p_1^L, is the change in market price resulting from a change in release

$$p_1^L = \frac{\partial p^*}{\partial q^t} = \frac{\rho}{I'+1} < 0 \tag{9.12}$$

which is constant and non-positive because $\rho<0$. Since the curve is downward sloping, the y-intercept, p_0^L, will occur where $q^t=0$, which is just the definition of equilibrium follower price when $q_k^t=0$. i.e.,

$$p_0^L = \frac{p_0 + \bar{c}_m I' + \rho f_m^t - \rho g_0}{I'+1} \tag{9.13}$$

The form of $p_L(q^t)$ is therefore:

$$\begin{aligned} p_L(q^t) &= p_0^L + p_0^{\prime L} q^t \\ &= \frac{p_0 + \bar{c}_m I' + \rho f_m^t - \rho g_0}{I'+1} + \frac{\rho q^t}{I'+1} \end{aligned} \tag{9.14}$$

which is linear with slope $\frac{\rho}{I'+1}$. The curve is downward sloping because $\rho<0$.

The profit from release can now be restated as:

$$Profit(q^t) = p_L(q^t) \times (q^t - f_h^t) \tag{9.15}$$

which is concave over q^t. Contract revenue may also be added, though this requires an assumption about the contract strike price. The form of $Profit(q^t)$ is of interest, particularly the marginal profit, which has two components. For a unit increase in release there will be a change in revenue generated from the additional unit of release as well as a change in revenue resulting from a different price being applied to the previous level of generation i.e.,

$$\begin{aligned} \frac{\partial Profit(q^t)}{\partial q^t} &= p_L(q^t) + \frac{\partial p_L(q^t)}{\partial q^t} \times (q^t - f_h^t) \\ &= p_L(q^t) + \frac{\rho(q^t - f_h^t)}{I'+1} \end{aligned} \tag{9.16}$$

The first term is the price associated with q^t and will be non-negative and non-increasing for increasing q^t, as per the earlier definition of $p_L(q^t)$. The second term is linear in q^t with slope $\frac{\rho}{I'+1}<0$ and is positive for $q^t<f_h^t$ and negative for $q^t>f_h^t$. For low q^t, $p_L(q^t)>\frac{\rho}{I'+1}$, while the reverse will occur for large q^t.

Consider a simple example with the hydro firm and two additional firms with marginal costs of c_1=1.5 and c_2=2.5, and contract quantities f_1=800 and f_2=1200. The parameters for demand are p_0=3, ρ=-0.001 and g_0=2000. The market parameters are I'=2, $\bar{c}_m = 2$ and f_m=2000. These are the same parameters used by Scott. The contract level for the hydro firm is f_h=500.

The case where hydro release is zero (q_k=0) is the same situation presented by Scott where there are only two firms. The equilibrium price is p_F^*=2.33 with total system generation of $g_F^* = g_m^*$=2666, which is comprised of g_1^*=1633 and g_2^*=1033. This solution is shown in Figure 9.2.

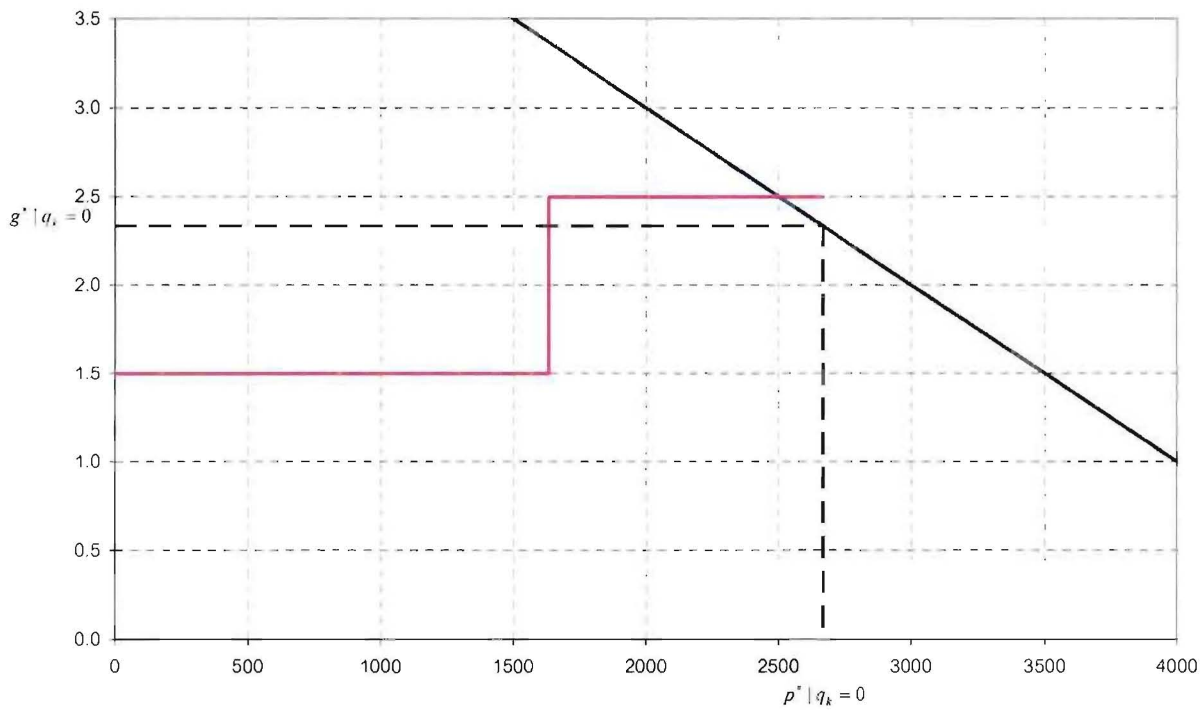


Figure 9.2: Equilibrium price and generation when q_k=0

For firm 2, $g_2^* < f_2$, so quantity $g_2^* - f_2$ is purchased by firm 2 from the spot market to satisfy its contract obligations, so a lower p^* will be desirable. Firm 1, on the other hand, has $g_1^* > f_1$, and is achieving a positive spot market profit, so a higher p^* will be preferable. The hydro firm's profit is calculated as $Profit(q') = \frac{7}{3}(0 - 500)$ = -1166, so for this particular release, the hydro firm would satisfy its contract obligation by purchasing the entire quantity off the spot market and incur a loss.

Now let q_k=1000. The demand curve is shifted to the left by 1000. The equilibrium price is p_F^*=2, with g_F^*=2000 comprised of g_1^*=1300 and g_2^*=700, as illustrated in Figure 9.3. Again, firm 1 has $g_1^* < f_1$, while $g_2^* > f_2$ for firm 2. The hydro firm is now making a positive spot profit of $Profit(q')$=2(1000-500)=1000.

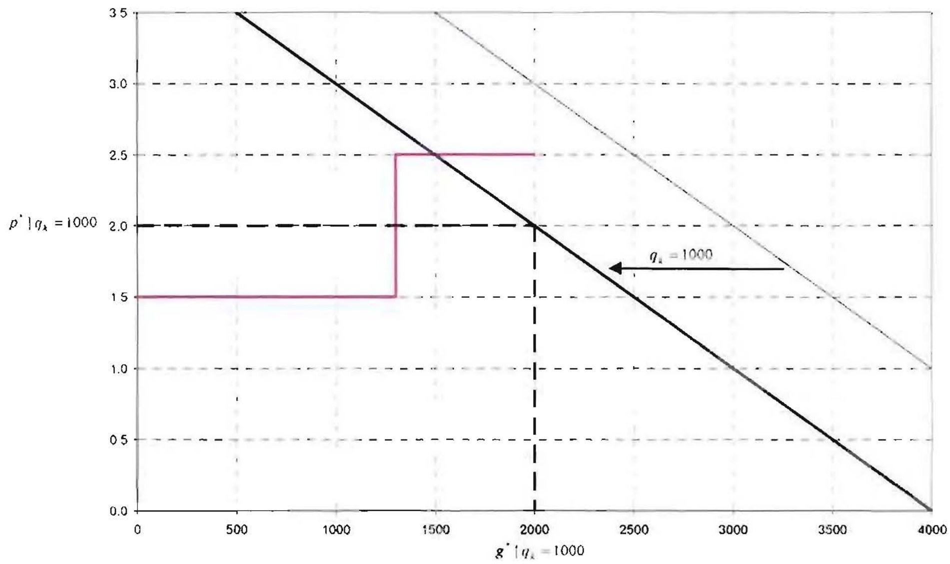


Figure 9.3: Equilibrium price and generation when q_k=1000

Evaluating $Profit(q')$ for a range of q_k' and for a fixed yields the profit curve shown in Figure 9.4. Contract revenue has been excluded from the profit calculations but would only result in the curves being shifted upwards by the contract revenue.

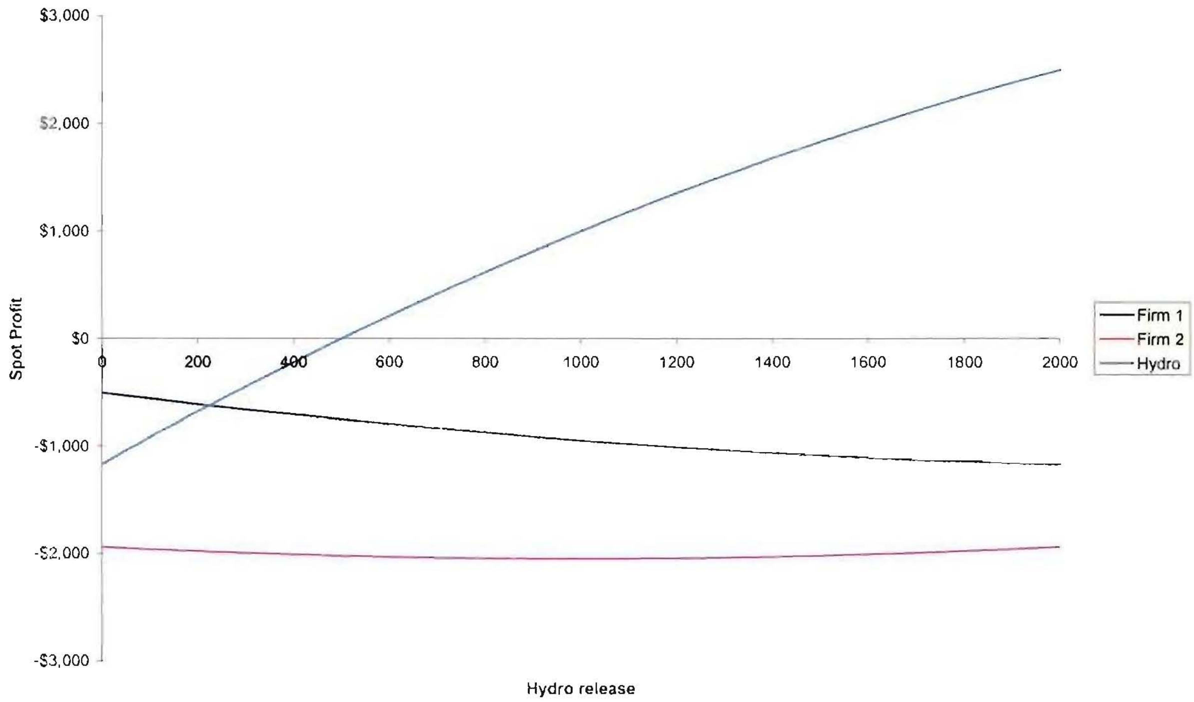


Figure 9.4: Firm profit levels (L demand, fixed marginal cost)

Hydro profit is concave and decreasing at a non-increasing rate, as discussed earlier. As q' increases, follower generation levels decrease linearly and, combined with the impact of q' on p_F^*, yield convex profit curves. Under the assumptions stated, if the benefit in a given period defined to be $Profit(q')$, algorithmic improvements would be possible because it is concave.

9.2.2 Constant elasticity demand

We now consider the case where demand is modelled by a constant elasticity demand curve. From Scott, if the price elasticity of demand is defined as:

$$\varepsilon = -\frac{\partial g}{\partial p}\frac{p}{g} \tag{9.17}$$

then the inverse market demand curve can be described as:

$$p(g_m) = p_0\left(\frac{g_m}{g_0}\right)^{-\frac{1}{\varepsilon}} \tag{9.18}$$

Ignoring the hydro firm, the equilibrium price is found by solving:

$$\left(1+\varepsilon I'\right)p^{*\varepsilon}-\varepsilon I'\bar{c}_m p^{*\varepsilon-1}-\frac{f'_m p_0{}^{\varepsilon}}{g_0}=0 \tag{9.19}$$

which is polynomial in p^* and can be solved numerically.

If the follower/leader framework described earlier is adopted here, the follower demand curve needs to be created by adjusting the market demand curve for the leader generation. There are two ways which this can be done. Firstly, as for the linear demand case, the reference generation level can be adjusted by q_k^t, giving:

$$p_F\left(g_F\right)=p_0\left(\frac{g_F}{g_0-q_k^t}\right)^{\frac{1}{\varepsilon}} \tag{9.20}$$

Because the slope of the demand curve is dependent on the generation level, the follower and leader demand curves will only have the same slope at the reference points $\left(p_o,g_0\right)$ and $\left(p_o,g_0-q_k^t\right)$. For $g_F\neq\left(g_0-q_k^t\right)$, $\partial p_F/\partial g_F\neq\partial p/\partial\left(g_F+q_k^t\right)$, so there will be an inconsistency between $p_F\left(g_F^*\right)$ and $p\left(g_m\right)$, where $g_m=g_F^*+q_k^t$.

To ensure that $p_F\left(g_F^*\right)=p\left(g_m\right)$, $p_F\left(g_F^*\right)$ must be horizontally displaced from $p\left(g_m\right)$ by q_k^t $\forall p_F$. The form of the follower inverse market demand curve which will ensure that $p_F\left(g_F^*\right)=p\left(g_m\right)$ is therefore:

$$p_F\left(g_m,q_k^t\right)=p_0\left(\frac{g_m-q_k^t}{g_0}\right)^{\frac{1}{\varepsilon}} \tag{9.21}$$

which is equivalent to a demand curve of the form:

$$g_F\left(p_m,q_k^t\right)=g_0\left(\frac{p_m}{p_0}\right)^{\varepsilon}-q_k^t \tag{9.22}$$

and the equilibrium price will be that which satisfies:

$$p_0\left(\frac{g_m}{g_0}\right)^{\frac{1}{\varepsilon}}-q_k^t=\frac{pf'_m}{\varepsilon I'\left(p-\bar{c}_m\right)+p} \tag{9.23}$$

Rearranging gives:

$$(1+\varepsilon I')p_F^{*\varepsilon} - \varepsilon I'\overline{c}_m p_F^{*\varepsilon-1} - \frac{f'_m p_0^{\ \varepsilon}}{g_0} = \frac{q_k^t p_0^{\ \varepsilon}(\varepsilon I'+1)}{g_0} - \frac{q_k^t p_0^{\ \varepsilon}\overline{c}_m}{g_0 p_F^*} \tag{9.24}$$

so the equation to be solved for each q_k^t is:

$$(1+\varepsilon I')p_F^{*\varepsilon} - \varepsilon I'\overline{c}_m p_F^{*\varepsilon-1} - \frac{f'_m p_0^{\ \varepsilon}}{g_0} - \frac{q_k^t p_0^{\ \varepsilon}(\varepsilon I'+1)}{g_0} + \frac{q_k^t p_0^{\ \varepsilon}\overline{c}_m}{g_0 p_F^*} = 0 \tag{9.25}$$

Note that setting q_k^t=0 yields the following polynomial:

$$(1+\varepsilon I')p_F^{*\varepsilon} - \varepsilon I'\overline{c}_m p_F^{*\varepsilon-1} - \frac{f'_m p_0^{\ \varepsilon}}{(g_0 - q_k^t)} = 0 \tag{9.26}$$

which is exactly the same as Equation 3.38 in Scott (1997), as we would expect, because demand is unaffected by the hydro firm. Once p_F^* has been determined, the equilibrium follower generation level is:

$$g_F^* = g_0\left(\frac{p_F^*}{p_0}\right)^{\varepsilon} - q_k^t \tag{9.27}$$

The individual firm generation levels are then calculated using Equation 3.41 in Scott (1997):

$$g_i^* = \frac{p_F^* - c_i}{\dfrac{p_F^*}{\varepsilon g_F^*}} + f_i \tag{9.28}$$

Consider again the example described earlier where there is a hydro firm and two follower firms with marginal costs of c_1=1.5 and c_2=2.5, and contract quantities f_1=800 and f_2=1200. The contract level for the hydro firm is f_h=500. The parameters for demand are p_0=3, g_0=2000, and ε=-3/2. The market parameters are I'=2, $\overline{c}_m = 2$ and f_m=2000. These are the same parameters used by Scott and in the linear demand example.

The case where hydro release is zero (q_k=0) is the same situation presented by Scott where there are only two firms. The equilibrium price is p_F^*=2.26 with total system generation of $g_F^* = g_m^*$=3057, which is comprised of g_1^*=2343 and g_2^*=714. This solution is shown in Figure 9.5.

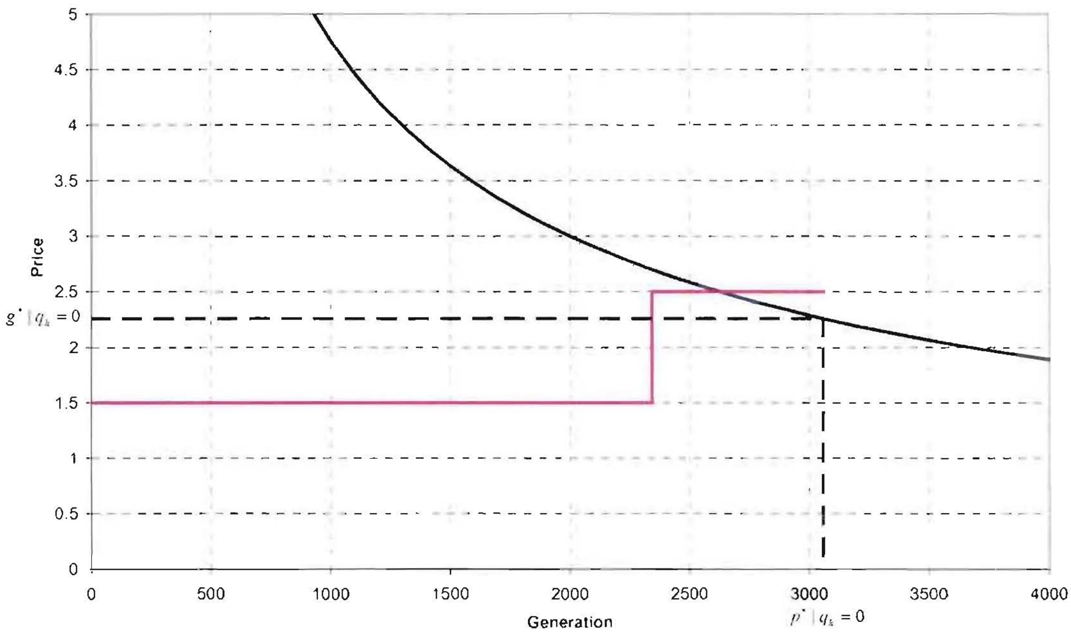


Figure 9.5: Equilibrium price and generation when q_k =0 (CE demand)

As for the linear demand case, $g_2^* < f_1$, and $g_1^* > f_1$. The hydro firm's profit is calculated as $Profit(q') = \frac{7}{3}(0-500)$ = -1166, so for this particular release, the hydro firm would satisfy its contract obligation by purchasing the entire quantity off the spot market and incur a loss.

Now let q_k=1000. The demand curve is shifted to the left by 1000. The equilibrium price is p_F^*=2, with g_F^*=2000 comprised of g_1^*=1300 and g_2^*=700, as illustrated in Figure 9.6. Again firm 1 has $g_1^* < f_1$ while $g_2^* > f_2$. The hydro firm is now making a positive spot profit of $Profit(q')$=2(1000-500)=1000.

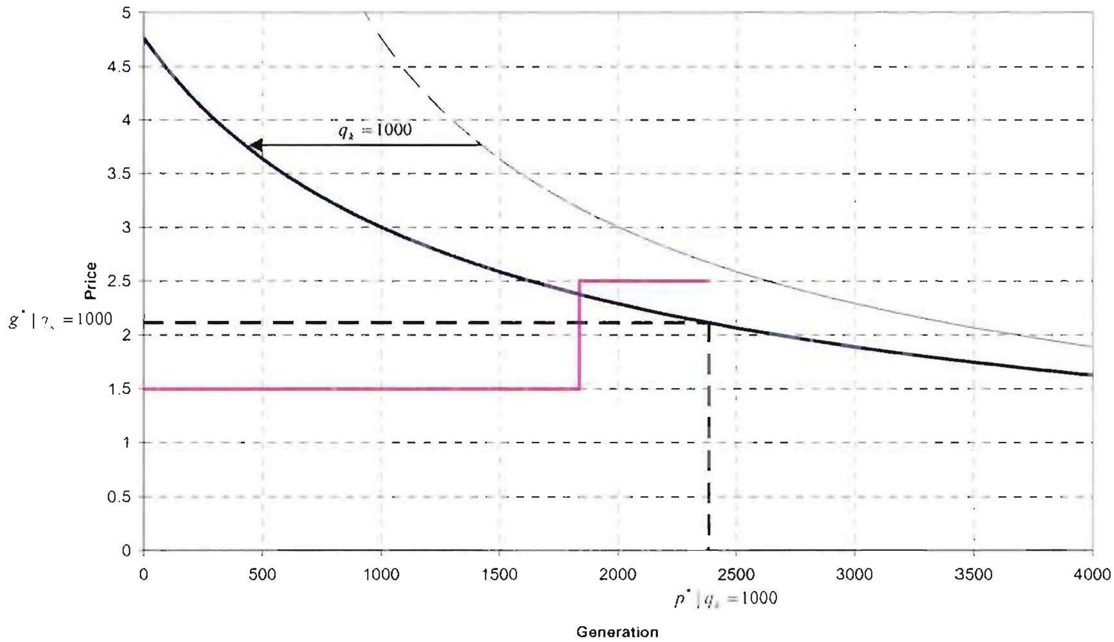


Figure 9.6: Equilibrium price and generation when q_k=500 (CE demand)

Evaluating $Profit(q')$ for a range of q_k' and for fixed 'follower' marginal costs yields the profit curve shown in Figure 9.7. Also shown is the spot profit curve derived using linear demand. The profit curves are concave over the range of feasible release levels and are similarly scaled for both the linear and constant elasticity demand curves.

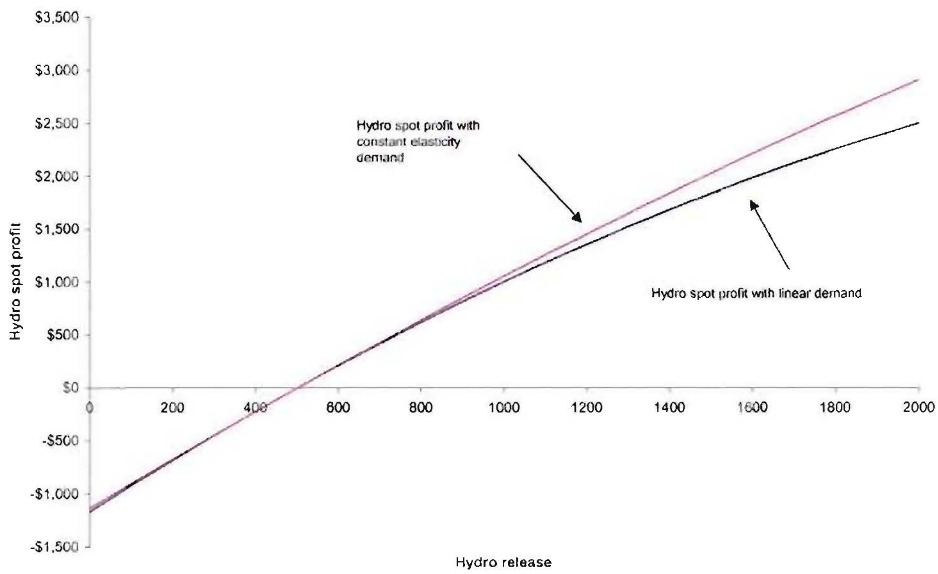


Figure 9.7: Hydro profit levels (CE demand, fixed marginal costs)

The market prices and generation levels associated with each hydro release are shown in Figure 9.8. When demand is assumed to be linear and the followers have fixed marginal costs, the price and generation curves are both linear, which was shown earlier. When a constant elasticity demand curve is used, the price curve has a flatter trajectory and is concave due to the demand curve and market generation curves being convex.

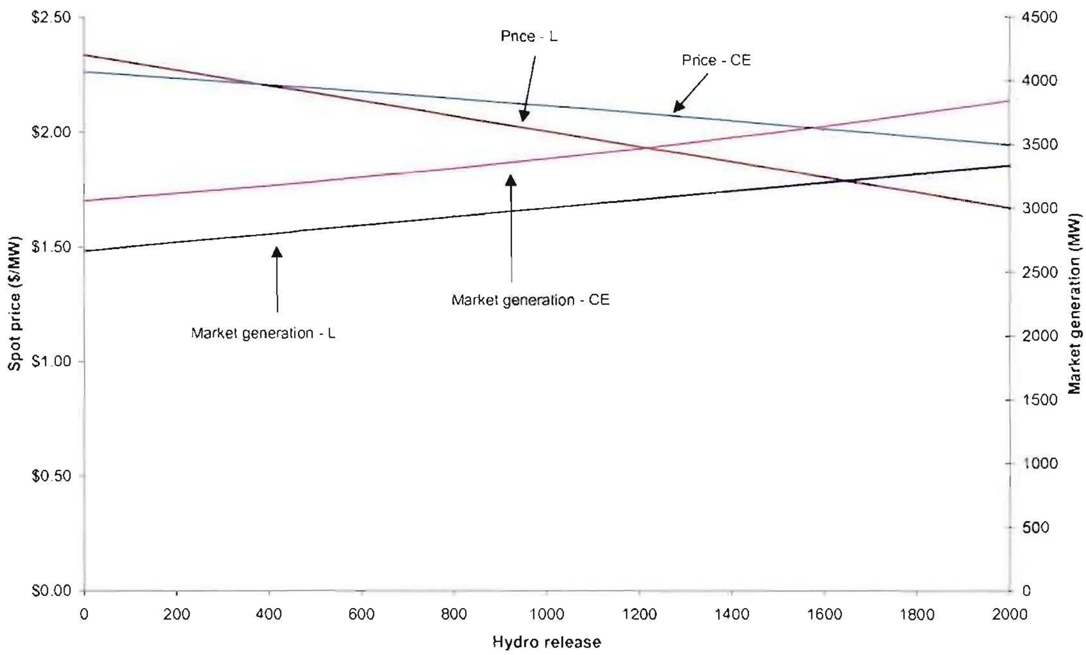


Figure 9.8: Spot price profiles (CE demand, fixed marginal cost)

9.2.3 Stepped supply curves and linear demand

Now consider the case where the follower firms have stepped supply curves. The previous analysis discussed the case where the marginal costs of each firm were fixed, which corresponds to generating on a particular step on the supply curve. Thus, a feasible generation level for a firm always corresponded to a single marginal cost. With each firm having a stepped supply curve consisting of vertical and horizontal segments, the equilibrium follower generation level will correspond to a single or multiple marginal costs, depending on whether the generation level corresponds to a vertical or horizontal segment of the supply curve.

The equilibrium price, p_k^*, associated with each discrete release level, q_k^l, must now be determined. One way to do this is to search all possible intersections of the follower firm supply curves until an equilibrium is found, and to do this for each q_k^t; Scott (1997) proved that if demand is linear, there is a unique equilibrium price. Each follower firm's supply curve is separated into horizontal segments and vertical segments, with D_i being the number of horizontal segments for firm *i* and E_i being the number of vertical segments for firm *i*. The horizontal segments are defined for a fixed marginal cost ($\hat{c}$) and over ranges of generation for each firm, and these bounds are denoted by the variables $\overline{h}$ and $\underline{h}$. The vertical segments are defined for a fixed generation level ($\hat{g}$) and over a range of marginal costs, and these bounds are denoted by the variables $\overline{v}$ and $\underline{v}$. Demand is linear and defined as $p(g_m) = p_0 + \rho(g_m - g_0)$. Each of the two firms is contracted to supply f_i, so the total level of 'follower' contracting is defined as $f_F = f_1 + f_2$.

An algorithm to find p_k^* is as follows:

For each *k* (discrete hydro release q_k^t)

    For $d1 = 1 \ldots D_1$ (set of horizontal segments for firm 1)

        $c_1 = \hat{c}_{d1}$

        For $d2 = 1 \ldots D_2$ (set of horizontal segments for firm 2)

$$c_2 = \hat{c}_{d2}$$

$$\overline{c}_m = (c_1 + c_2)/2$$

$$g_i = f_i + \frac{3c_i - p_0 - 2\overline{c}_m - \rho f_F + \rho(g_0 - q_k^t)}{3\rho} \quad \text{for } i=1,2$$

if $\overline{h}_{d1} > g_1 \geq \underline{h}_{d1}$ and $\overline{h}_{d2} > g_2 \geq \underline{h}_{d2}$ (then solution is admissible)

$$p_k^* = \frac{p_0 + 2\overline{c}_m + \rho f_F - \rho(g_0 - q_k^t)}{3}$$

        end *d2*

        For $e2 = 1 \ldots E_2$ (set of vertical segments for firm 2)

$$g_2 = \hat{g}_{e2}$$

$$g_1 = f_1 + \frac{c_1 - p_0 - \rho f_1 + \rho(g_0 - q_k^t - g_2)}{2\rho}$$

$$p = \frac{p_0 + c_1 + \rho f_1 - \rho(g_0 - q_k^t - g_2)}{2}$$

$$c_2 = p - \rho(g_2 - f_2)$$

if $\overline{h}_{d1} > g_1 \geq \underline{h}_{d1}$ and $\overline{v}_{e2} > c_2 \geq \underline{v}_{e2}$ (then solution is admissible)

$$p_k^* = p,\ g_{1k}^* = g_1,\ g_{2k}^* = g_2$$

        end *e2*

end *d1*

For *e1* $= 1 \ldots E_1$ (set of vertical segments for firm 1)

For *d2* $= 1 \ldots D_2$ (set of horizontal segments for firm 2)

$$g_1 = \hat{g}_{e1}$$

$$g_2 = f_2 + \frac{c_2 - p_0 - \rho f_2 + \rho\left(g_0 - q_k^t - g_1\right)}{2\rho}$$

$$p = \frac{p_0 + c_2 + \rho f_2 - \rho\left(g_0 - q_k^t - g_1\right)}{2}$$

$$c_1 = p - \rho\left(g_1 - f_1\right)$$

if $\bar{v}_{e1} > c_1 \geq \underline{v}_{e1}$ and $\bar{h}_{e2} > g_2 \geq \underline{h}_{e2}$ (then solution is admissible)

$$p_k^* = p,\ g_{1k}^* = g_1,\ g_{2k}^* = g_2$$

end *d2*

For *e2* $= 1 \ldots E_2$ (set of vertical segments for firm 2)

$$g_1 = \hat{g}_{e1}$$

$$g_2 = \hat{g}_{e2}$$

$$p = p_0 - \rho\left(g_0 - q_k^t - g_1 - g_2\right)$$

if $\bar{v}_{e1} > p \geq \underline{v}_{e1}$ and $\bar{v}_{e2} > p \geq \underline{v}_{e2}$ (then solution is admissible)

$$p_k^* = p,\ g_{1k}^* = g_1,\ g_{2k}^* = g_2$$

end *e2*

end *e1*

end for *k*

An alternative method is to find the Cournot equilibrium by solving a MCP formulation of the problem. The MCP solution can then replace the rather laborious process described above i.e.,

For each *k* (discrete hydro release q_k^t)

Find p_k^* and g_{ik}^* by solving a MCP formulation

end for *k*

The MCP formulation contains the three equations used in the previous analysis. Firstly, total follower generation is defined as the sum of the generation of each station (*j*) owned by each firm (*i*):

$$\sum_i \sum_j g_{ij} - g_F = 0 \tag{9.29}$$

where the g_{ij} variables have upper bounds corresponding to the station capacities. The price resulting from total follower generation and hydro release can then be defined, and represented in constraint form as:

$$p_0 + \rho\left(g_F + q_k^t - g_0\right) - p = 0 \tag{9.30}$$

Each firm operates at a level where the marginal cost of generation is equal to the marginal revenue:

$$c_{ij} - \rho\left(g_{ij} - f_i\right) - p = 0 \tag{9.31}$$

This constraint and that which defines g_F are expressed as ≥ constraints in the MCP formulation.

Consider now an example described earlier where there is a hydro firm and two follower firms with contract quantities f_1=800 and f_2=1200. Each firm has two stations. For firm one, the marginal cost and capacity vectors are (1.5,3.5) and (700,1000), and for firm two they are (2.5,5) and (1000,1000), respectively. The parameters for demand are p_0=3, g_0=2500, and ρ=-0.001. This is essentially the same illustrative data discussed by Scott (1997, p 48); in his example, g_0=2500 and upper bounds on each firms most expensive station were not specified. The contract level for the leader is set at f^t=500 and the maximum release as $\overline{q}^t$=1000. The spot price and leader profit curves for the range of q^t are shown in Figure 9.9.

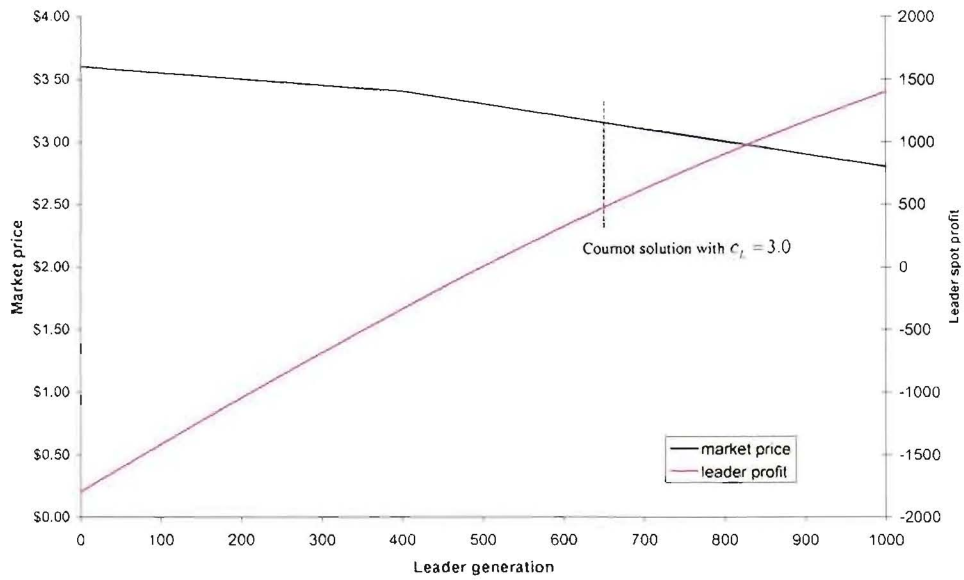


Figure 9.9: Spot price and leader profit (linear demand)

When q'=0, the Cournot solution is g_1=900 and g_2=1000 which results in a price p^*=\$3.60. The price curve is non-increasing and in this example consists of two linear segments. As q' increases from 0 to 400, g_1 decreases at the rate of $0.5q'$ and g_2 remains fixed at 1000. This corresponds to the first (linear) segment in the spot price curve. Leader profit remains negative over this range of generation levels as $q' < f'$. At q'=400, the follower solution is g_1=700 and g_2=1000. These follower generation levels remain the same for $400 \leq q' \leq 1000$. With follower generation levels essentially fixed, the price decreases at a higher rate for $q' \geq 400$ than when $q' \leq 400$, corresponding to the second linear segment in the spot price curve. The leader profit curve is piecewise quadratic due to the price curve being comprised of two linear segments and maximum profit is made where $q' = \overline{q}'$. As a point of comparison, the Cournot solution was calculated where the leader used a marginal cost of 3 for the entire range of output. The leader generation was 650 (g_1=700 and g_2=1000), which lies between f' and $\overline{q}'$, while p^*=3.15.

9.2.4 Stepped supply curves and constant elasticity demand

MCP can again be used to handle constant elasticity demand. The formulation is the same as for the linear demand case aside from the different definition of the demand curve. As discussed earlier, to ensure that leader generation has the desired impact on the follower demand curve, the price constraint is defined as

$$g_F - g_0\left(\frac{p}{p_0}\right)^{\varepsilon} + q'_k = 0 \tag{9.32}$$

The same analysis performed using linear demand was performed using constant elasticity demand. The spot price and leader profit curves are illustrated in Figure 9.10. The curves are almost identical to the case with linear demand due to the fact that the linear and constant elasticity demand curves are very similar for generation/price values around the reference points. Increasing leader generation shifts both curves back by the same amount, so the curves remain close for generation/prices around the adjusted reference points.

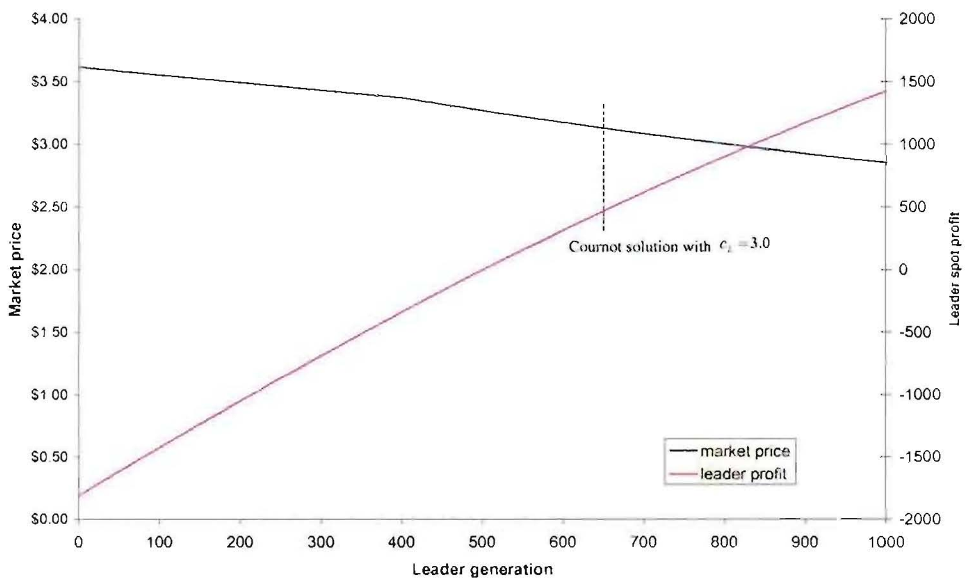


Figure 9.10: Spot price and leader profit (CE demand)

The Cournot solution when the leader is treated as a player (with marginal cost = 3) is p^*=3.13 and the firm's generation is 646MW. This is almost identical to the linear demand case which had p^* 3.15 and generation of 650MW. Note that when q'=500 the solution is identical to that presented in Scott (1997).

9.3 Managing hydro and thermal plant

In the discussion thus far, the quantity of electricity supplied by the hydro firm was equal to the electricity generated from reservoir release. We now consider the case where the hydro firm is a price setter and owns an additional thermal plant, and show that it is possible to jointly optimise hydro and 'thermal' generation without much extra additional computation.

The firm must now determine the total electricity it will offer to the market, and this will be comprised of hydro release (q) and thermal generation (G), where:

$$G = \sum_j g_j \tag{9.33}$$

Reservoir release in a given period is bounded, and thermal generation may be bounded.

Recall that in , revenue from release was defined as the sum of contract revenue and spot revenue in the period:

$$B(q) = p_F f + p_S(q - f) \tag{9.34}$$

where p_F is the contract strike price and p_S is the spot price, which is dependent on the demand curve, fringe firms, and level of release (the t subscripts have been dropped). The other players were assumed to be price takers, so the residual demand curve faced by the firm was derived by subtracting fringe supply from the demand curve, whether it be fixed, linear or of constant elasticity. The spot price for a particular release could then be read directly off the residual demand curve.

The above definition of benefit does not include a cost of generation because reservoir release is assumed to have no direct variable cost. Now, though, the price is a function of the firm's total generation but the cost is only dependent on the firm's total thermal generation (G). Thus we have:

$$B(q, G) = p_F f + p_S(q + G - f) - C(G) \tag{9.35}$$

where $C(G)$ is a function describing the cost of thermal generation. If the firm's thermal generation is represented as a stepped supply curve, then $C(G)$ will be convex if the units are dispatched in order of increasing marginal cost.

Ignoring storage issues, hydro release is the most desirable because it has no explicit cost, so for some spot price p_S, supplying a unit of electricity will yield a gross profit of (p_S-0), which will always be larger than the gross profit derived from supplying the electricity from the thermal source j at marginal cost c_j (with gross profit $p_S - c_j$). While a unit of electricity generated using hydro will always yield a greater gross profit than that supplied from a thermal station, hydro release also reduces the storage level and hence affects the way in which the water can be used later in the planning horizon. Thus, a unit of hydro generation affects both wealth and storage, while a unit of thermal generation only affects wealth. Owning an additional 'unconstrained' generation plant will increase the benefit associated with each hydro generation level. If the thermal plant is constrained in some way (e.g. lower bounds on generation), then the benefit from release may actually be worse compared to the case where the plant is not owned. This can occur because the operating cost of the 'constrained on' plant can exceed the spot price.

The form of the benefit function will differ now because the firm can choose between hydro and thermal generation sources. Assuming that thermal stations are dispatched in order of lowest to highest marginal costs, and ignoring intertemporal scheduling issues, the firm's optimal generation level for a particular wealth and storage level can be derived in a fairly straight forward manner.

In terms of implementation, recall that reservoir release (generation) is discretised at K points, so q_k is the k'th discrete release level ($k \in K$). Thermal release is also discretised at l points, so G_l is the l'th discrete generation level ($l \in L$). The simplest algorithm to find the optimal values of (q_k, G_l) for some $(w^t, s^t) \in W^t \times S^t$ is to evaluate all combinations of (q_k, G_l) so as to maximise expected utility i.e., $g^t(w^{t+1}, s^{t+1})$.

Although $B^t(q, G)$ only needs to be computed once in each period, the inclusion of thermal generation has a similar effect on computational effort to increasing the state space. However, if $g^t(w^{t+1}, s^{t+1})$ is non-decreasing in both storage and wealth, a significantly more efficient process can be used to optimise hydro and thermal release.

This process utilises the fact that a unit of hydro generation affects both wealth and storage, but a unit of thermal generation only affects wealth.

In the purely hydro case, the optimal release for a particular (w^t, s^t) pair was determined by evaluating $g^t(w^{t+1}, s^{t+1})$ for each discrete release (q_k). For a particular q_k, $B(q, G)$ can be calculated given q_k (effectively fixed) and over the range of possible thermal generation levels. Let G^{*k} denote the optimal thermal generation given that release is q_k. If $g^{t+1}(w^t, s^t)$ is non-decreasing and G^{*k} maximises $B(q_k, G^{*k})$, then G^{*k} will also maximise $g^{t+1}(w^t, s^t)$. Observe that the maximisation of $B(q_k, G^{*k})$ can be performed independently of (w^t, s^t), and the calculations only need to be performed once in each period if q and G are independent of the levels of storage and wealth. With these conditions satisfied, a single benefit function can be derived and used in exactly the same manner as for the hydro case to optimise q_k for a given (w^t, s^t) pair.

In , $B^t(q)$ had a discontinuous 'saw-tooth' shape. The slope of each kink (tooth) was equal to the marginal cost of the marginal fringe thermal station. When thermal generation is considered, the benefit function will have the same general shape, but the slope will also reflect the marginal cost of the firm's thermal generation. In the purely hydro case, the cost of production was zero, so kinks in the benefit curve corresponded to release levels which forced a change in the marginal station. With thermal production, the benefit function will also have kinks corresponding to changes in the cost of the firm's thermal generation. The kinks corresponding to a change in the firm's marginal cost are not nearly as large as those caused by a change in marginal fringe station because the latter affects the profit generated by the firm's total generation whereas the former only affects the cost of a single unit of generation.

Let $B(q_k, G)$ refer to the function which describes the benefit from generating G given a fixed value of q_k. Let $G^*(q_k)$ be the firm's profit maximising thermal generation given hydro generation q_k. Let $B(q, G^*(q))$ refer to the function describing the total benefit to the firm given hydro generation q and the associated optimal thermal generation $G^*(q)$. In order to find $G^*(q_k)$ that maximises $B(q_k, G^{*k})$ for a particular q_k, a search over the entire range of G is not required. All that is required is a comparison of the values of G which correspond to the 'point' of a 'tooth' in the benefit function. If the firm owns J thermal stations and the fringe consists of J_F stations, then there will be a maximum of J_F breakpoints in the residual demand curve, and hence a maximum of $J + J_F$ breakpoints in $B(q_k, G)$. As demand increases, the number of breakpoints in the residual demand curve will increase, and as q_k increases, the number of breakpoints in $B(q_k, G)$ will decrease. Regardless, with a price taking competition, the number of breakpoints to evaluate for each is q_k is still a relatively small number, so the computational effort required to derive $B(q_k, G)$ would not overly exceed that required to derive benefit function for the regulated and deregulated cases described in earlier chapters.

9.4 Conclusion

Chapters 4-8 described and illustrated SUMDP for a reservoir operating in 'regulated' and 'deregulated' environments. Alternative benefit functions can be envisaged, and a few of those were discussed in this chapter, namely making a different assumption about the response of other players to the release decision from the hydro firm, and where the offer from the hydro firm incorporates a thermal generation owned by the same firm. These benefit functions are a function of release and can be derived independently from SUMDP. Once derived though, they could be incorporated in SUMDP with the value function used to derive the optimal release levels in the same way as illustrated for the earlier cases.

In the next chapter, SUMDP is discussed for stochastic route choice problems, which have similar characteristics to the reservoir management problem, and for which there are few approaches discussed in the literature which produce dynamic solutions, consider uncertainty, and consider risk.

Chapter 10

SUMDP and Stochastic Route Choice

10.1 Introduction

For the medium-term reservoir management problem, SUMDP corresponded to finding an optimal dynamic (or non-stationary) route (or path) through a discrete directed acyclic stochastic network. Planning problems in the domains of energy, finance, transportation, telecommunications, and project management often have a similar structure. In essence, the problem is this: given an origin node, a destination node, and a number of intermediary nodes all linked by arcs with uncertain lengths, what is the 'shortest' route from the origin to the destination. These problems are often referred to as stochastic route choice (SRC) problems. The aspect of interest is that, in the literature, SRC formulations typically have separable objectives where an expected value is maximised/minimised at each node/stage. The optimal routes are therefore 'risk neutral', which may not always be appropriate.

As discussed, SUMDP maximises a non-linear function of the outcomes associated with each decision made during the planning horizon. For a SRC problem, this is equivalent to the objective being a function of all the arcs in routes, and the optimal route being dynamic (or non-stationary). For some specific forms of arc uncertainty and utility, SRC problems can reduce to a deterministic problem (Loui, 1983). Deterministic dynamic programming can be applied because the optimal decision at each node is independent of how the node was arrived at (Bellman's principle of optimality). For less restrictive assumptions about utility and uncertainty, though, the objective remains a non-separable function of the arcs that comprise a route. Therefore, Bellman's principle of optimality does not hold, and complete enumeration is the only technique that will guarantee a globally optimal solution (Loui 1983, Bard & Bennett 1991).

There are a relatively small number of studies which address SRC problems where the DM has a non-linear preference towards terminal outcomes, and the solutions are static rather than dynamic. Static solutions are determined prior to any realisation of uncertainty, while dynamic solutions allow the DM to adjust the route once some uncertainty has been realised, while still maximising the same objective[8]. While static solutions are appropriate in contexts where a `one-shot' route is required (e.g., route choice for hazardous materials), there are many situations where a dynamic solution appears to be more appropriate. For example, Bard & Miller (1989) describe a project selection problem with a budget constraint and an objective of maximising a non-linear utility function. The budget can be allocated to projects in order to improve the distribution of project completion times. The solution from their algorithm is static, in that not only are the projects selected prior to the outcomes of the project times, but so are the budget allocations. A more appropriate solution to that problem would have the project selection and budget allocation decisions dependent on the time taken to complete earlier projects. Moreover, multiple projects might be optimal at a decision node, depending on the time taken to complete the previous projects and remaining budget. A problem of this type is illustrated in a later section. To the author's knowledge there are no methods which describe an optimal, dynamic, utility maximising solution to the SRC problem, a problem which has been described as "notoriously intractable" in Murthy & Sarkar (1997, p. 227).

In Section 10.2, the SRC problem is introduced. A simple SRC problem is solved using SDP to illustrate how a non-optimal (static) solution can be produced when the objective is non-linear. The literature is also reviewed. In Section 10.3, a SUMDP

[8] An alternative rationale is to assume that the DM's preferences change as the route is followed.

approach to the SRC problem is presented and illustrated with two example problems. Some extensions/variations to the SUMDP approach are presented in Section 10.4 in the context of specific problems, and a summary is given in Section 10.5.

10.2 Motivation

In this section, the route choice problem is introduced for a directed acyclic stochastic network. Let *I* be a finite set of nodes and $K(i,j)$ be a finite set of directed arcs each linking a pair of nodes (i,j) where $(i,j) \in I$ and *j*>*I*. The set of successor nodes (*j*) to node *I* are defined in the set $j \in J(i)$. The length of each arc, t_k, is assumed to be an independent random variable with some known distribution and there may be multiple arcs connecting a pair of nodes. The optimal route through the network (from 1 to *I*) is that which minimises the length of the route, and when uncertainty is present, with no risk aversion, minimises the expected route length.

Stochastic dynamic programming is one technique which can be used to find the optimal route through this network if the objective is to minimise the expected route length, as described in any standard operations research text (e.g., Daellenbach et. al. (1983)) or dynamic programming text (e.g., Denardo (1982)). Let $f(i)$ be the minimum expected route length from node *i* to node *I*; this is the cost-to-go function. It is easy to envisage problems with multiple successor nodes and arcs that skip nodes (illustrated in a later section). As long as the network is acyclic, these scenarios can be handled by SDP, and the SUMDP approach described earlier. The optimal decision at each node can be found by evaluating the following dynamic programming recursion:

SRC1 $$f(i) = \min_{j,k|j \in J(i), k \in K(i,j)} \mathrm{E}\left[t_k + f(j)\right] \qquad \forall i \neq I \quad (10.1)$$

where $f(I) = 0$. Starting from the penultimate node, the recursion is solved for each successive node i.e., from *i*=*I*-1 to *i*=1. The decision made at each node is the arc to 'travel' on, and hence what the successor node (*j*) will be.

Consider the network in Figure 10.1 which describes a network (project) consisting of 4 arcs (tasks). The nodes represent points at which tasks are completed and the next task is selected. The time taken to complete a task is uncertain, though it is assumed here that the nature of the uncertainty is known. A DM must determine the sequence of tasks which satisfies her objective; either task 1 or 2 is selected, then either task 3 or 4 is selected.

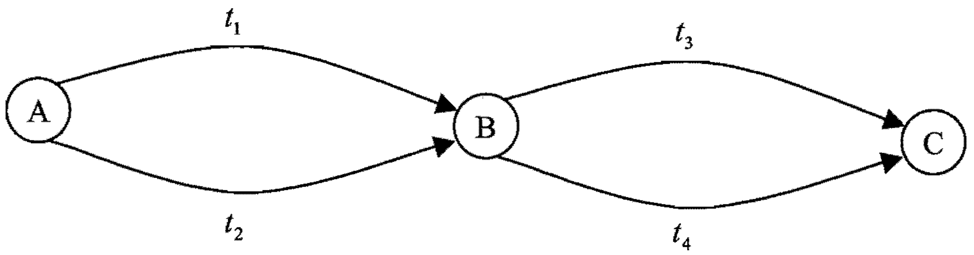


Figure 10.1: Network with stochastic travel times

In keeping with the simplicity of the network, the arc lengths are assumed to have discrete and finite probability distributions with two outcomes, v_k and w_k. The probabilities of these outcomes occurring are α_k and β_k, respectively, where $0 \le \alpha_k \le 1$ and $\beta_k = 1 - \alpha_k$. The expected length of arc k can therefore be calculated as $\bar{t}_k = \alpha_k v_k + \beta_k w_k$. Table 10.1 contains the task length data (in units of weeks, say) and the associated probabilities for this example.

From A to B		From B to C	
v_1=1	α_1=.4	v_3=7	α_3=.1
w_1=4	β_1=.6	w_3=6	β_3=.9
v_2=2	α_2=.3	v_4=8	α_4=.6
w_2=3	β_2=.7	w_4=4	β_4=.4

Table 10.1: Travel times and probabilities

For the arc data described here, the expected arc lengths are $\bar{t}_1$=0.4(1)+0.6(4) = 2.8, $\bar{t}_2$=2.7, $\bar{t}_3$=6.1, and $\bar{t}_4$=6.4. When the DM 'arrives' at node B (implying that either task 1 or 2 has been completed), either task 3 or 4 must be selected. Using the objective of minimising the expected project length, arc 3 is preferred to arc 4, and hence is the optimal arc to select from B. At node A, the DM can select from tasks 1 and 2, with the latter having the lower expected length. Thus, the solution to problem **SRC1** is route (2,3) which has an expected completion time of 8.8 (weeks).

Consider now a scenario where there is a time limit on the project length and this is reflected by a 'decreasing deadline' utility function of the form:

$$U(T_C) = \begin{cases} a_0 - a_1 T_C & 0 \le T_C \le d \\ 0 & T_C > d \end{cases} \tag{10.2}$$

where a_0, a_1 and d are all positive (see Figure 10.2 where a_0=100, a_1=5, and d=8). If T_C is greater than d hours, the project is deemed to have no value, reflected by zero utility. Equivalently, the negative slope of $U(T_C)$ can reflect the increasing reward for completing the project earlier than *d*. Note that the utility function could be in units of profit. The key issue is that the tasks do not contribute to the profit in themselves, it is the total time to complete 'the project', or all the tasks, which determines the profit. It is worth noting here that the utility functions discussed in the context of SRC problems (e.g. Bard and Bennett, 1991; Murthy and Sarkar, 1997) tend to have an economic interpretation, and are devoid of probabilistic interpretations.

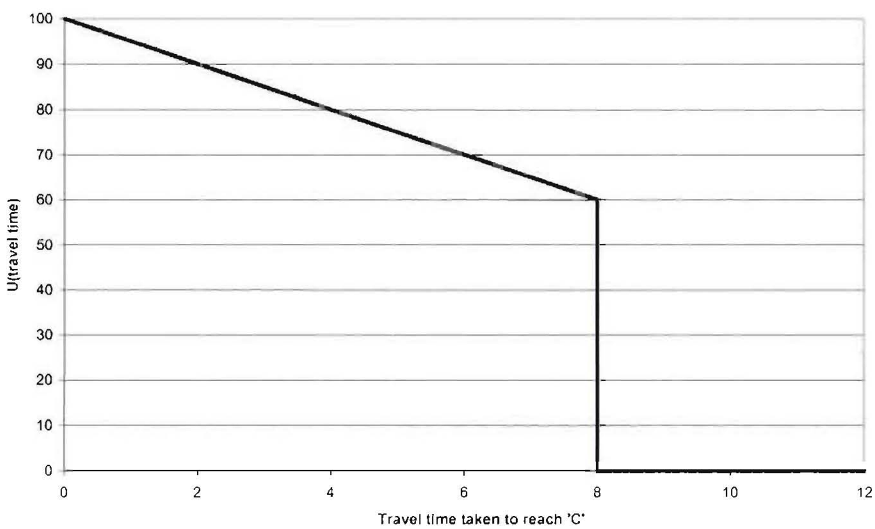


Figure 10.2: Decreasing deadline utility function

If **SRC1** is applied to this problem then the optimal arc at each node is that with the maximum expected utility. For node A, $E[U(t_2)] > E[U(t_1)]$ and for node B $E[U(t_3)] > E[U(t_4)]$, so the route that maximises the sum of the expected utilities is (2,3), the same as that which minimised the sum of the expected task lengths (and the expected project length). However, the DM wishes to select a route from A to C which maximises the expected utility associated with the *total* travel time, which depends on all the arc lengths in the optimal route. As shown in the introductory chapters, when the objective is defined this way, the optimal solution (in this case a route through a network) must be determined by considering the expected utility of this total time. The objective is therefore no longer additive, and the problem can not be solved using the dynamic programming formulation of **SRC1**.

One way to determine a utility maximising solution is to use a process of complete enumeration. This involves calculating the expected utility associated with every route in the network. Let p be a route from A to C and P be the set of all p, so P={(1,3), (1,4), (2,3), (2,4)}. For some p, the time taken to reach node C, T_C, is

$$T_C = \sum_{k \in p} t_k \tag{10.3}$$

where T_C is a random variable due to t_k being random. The optimal route is that which maximises the expected utility of the total travel time i.e.,

SRC2

$$\max_{p \in P} \; \mathrm{E}[U(T_C)] \tag{10.4}$$

For the example network, evaluating $\mathrm{E}[U(T_C)]$ for all p yields expected utilities of 25.8 (route 1-3), 26.4 (route 1-4), 16.2 (route 1-3), and 26.6 (route 2-4), so the optimal route for problem **SRC2** is (2,4). Routes (2,4), (1,3), and (1,4) are all preferred to (2,3), which was the optimal solution to problem **SRC1**. The solutions to both **SRC1** and **SRC2** are static in the sense that the task selected at B can not be adjusted depending on the length of task 2, which is the arc selected at A. **SRC1** is a DP, and the solution to **SRC1** is dynamic in the sense that the optimal arc is a function of the node, even though it wasn't apparent in the example. The drawback with complete enumeration, though, is that in addition to the optimal routes being static, the number of routes increases exponentially in the number of nodes. If dynamic routes were also included in P, the number of routes would quickly become intractable for even a small number of nodes. (Techniques have been developed for reducing the number of arcs in the network, although in the context of finding static solutions. These are discussed in the next section).

Considerable attention has been given to solution methods for deterministic and stochastic networks, with the latter problem being more complex and difficult to solve. Solution approaches are varied e.g. Markov decision processes, stochastic programming, and stochastic dynamic programming (Dror, 1993). When stochasticity is considered, the objective of most approaches is to minimise or maximise an expected value, which implies that the DM is risk neutral. The techniques that consider the DM's attitude to risk, usually reflected by some form of utility function, rely on Monte Carlo simulation or some similar form of scenario generation/evaluation to evaluate the DM's utility. Approaches such as these are required because utility is a function of *all* the decisions, so the objective is no longer additive, and Bellman's principle of optimality does not hold.

There are a relatively small number of studies which focus on the SRC problem where the DM has some preference or attitude towards the distribution of terminal outcomes. For some specific forms of arc uncertainty and utility, SRC problems can reduce to a deterministic problem (Loui, 1983) and can be solved using dynamic programming where the optimal decision from a node is independent of how the node was arrived at (Bellman's principle of optimality). For more general forms of utility and uncertainty, though, the objective remains a non-separable function of the arcs that comprise a route, so Bellman's principle of optimality does not hold, and complete enumeration is the only technique that will guarantee a globally optimal solution (Loui (1983), Bard & Bennett (1991)).

The concept of risk/utility rarely appears in the general SRC literature, even when uncertainty is a central issue (though implied risk attitudes are usually mentioned in the few papers that consider non-linear utility functions). Loui (1983), Bard & Bennett (1991), and Murthy & Sarkar (1997) all state that to determine an optimal static utility maximising solution **(SRC2)** requires complete enumeration of all possible routes. As an alternative, algorithms have been developed which reduce the size of the network by removing arcs. Eiger et. al. (1985) present an O(n^2) algorithm which produces a static solution when the DM's utility function is linear or exponential. Mirchandani & Soroush (1985) extend this work to handle a quadratic utility function. Bard & Bennett (1991) use Monte Carlo simulation and stochastic dominance rules to arrive at a set of "attractive" solutions. Murthy & Sarkar (1996, 1997, and 1998) developed rules for removing arcs when it can be proved that any route from a given node to the destination node will not be optimal. These rules are derived from the form of the uncertainty of the arc lengths and the form of the utility function (a decreasing deadline in Murthy & Sarkar (1997) and piecewise-linear in Murthy & Sarkar (1998)).

While the performance of some of these approaches is impressive, the major drawback is that the solutions are static in an environment where the uncertainty is dynamic. Static solutions to SRC problems are determined prior to any realisation of uncertainty, whereas dynamic solutions allow the DM to adjust the route once some uncertainty has been realised. While static solutions are certainly appropriate in many situations (e.g. route choice for hazardous materials), there are many contexts where dynamic updating is more consistent with a DM's behaviour and/or the decision making environment (e.g., reservoir planning, capacity planning, and many delivery/pickup problems) yet few published approaches for finding solutions (Murthy & Sarkar, 1997). The few techniques that address the dynamic stochastic shortest route problem require some restrictive assumptions about the form of the problem, as discussed below.

Dynamic (or time adaptive) solutions to SRC problems refer to the case where the DM can adjust the route having experienced some uncertainty during the trip. Hall (1983) introduced the concept of adaptive route choice, which he defines as the "process of adjusting route choice to information learned on the day of travel" (p. 203). He discusses adaptive route choice from a practical perspective, reporting on a number of experiments (using students) which investigated the way in which different types of information affected the traveller's expectations of travel times. Following from this work, Hall (1986) describes a dynamic programming formulation for the SRC which produces a dynamic solution when travel times are dependent on the time of arrival at a node. The network is divided into K stages, and a maximum of K decisions can be taken during the trip. The minimum expected final arrival time, having arrived at node n at time μ_n and made k arc selections is defined as $T_k^*(n, \mu_n)$. The state transition is defined as

$$T_k^*(i, \mu_i) = \min_{j \in N(i)} \int f_{ij}(t \mid \mu_i) T_{k+1}^*(j, t) dt \qquad \forall i \neq I \quad (10.5)$$

where $N(i)$ is the set of successor nodes (including multiple arcs) and $f_{ij}(t \mid \mu_i)$ is the probability density function for the arrival time at node j given that the traveller arrives at node i at time μ_i. He illustrates this technique when arc travel times have exponential probability distributions and produces an analytical solution for a small example problem.

A different approach is taken by Psaraftis & Tsitsiklis (1993), who consider a shortest route problem when the DM faces uncertain arc times, modelled by a Markov process, upon arrival at a node. The objective is to minimise the expected cost of traversing the network, and the decision process involves deciding whether to remain at a node to wait for a more promising future state or to travel along one of the arcs, and if so, determining which arc (node) to travel on (to). They discuss their approach in the context of ship routing. More recently, Pretolani (2000) described how hypergraph methods can be used to find dynamic solutions to stochastic acyclic networks with time-dependent arc lengths, which requires that the arc length distributions be discrete and the arc lengths positive. Although the accumulated time at a node is considered when determining the optimal hyperarcs, the objective must be additive for the hyperarc "weights" to be added.

Vehicle Routing Problems consist of determining a route for one or more vehicles which, starting from one or more origin nodes, ensures that the vehicle(s) visit(s) some set of nodes such that the route(s) satisfies some set of side constraints (e.g., demand) and maximises some objective(s). There has been considerable research on this problem and its variations, though there are relatively few approaches which handle issues relevant to this work, namely stochastic and/or time dependent arc times. Malandraki & Daskin (1992) consider the VRP when travel times are deterministic but dependent on the arrival time at a node. The travel time dependency is modelled using a step function, with the width of a step indicating the range of departure times over which a given travel time applies. They formulate the problem as a MILP. Laporte et. al. (1992) consider the VRP with stochastic travel and service times, and use chance constrained programming and stochastic programming with recourse to determine optimal static routes for *m* vehicles.

Risk has been incorporated into the transportation planning of hazardous materials (HM). Route choice problems for HM is a problem faced by many industrial countries especially with recent regulatory developments. Ashtakal & Eno (1996) describe a HM route choice model with an objective of minimising the impact of a spill on the population and environment. In this context, they define risk as the probability of an accident, multiplied by the consequence of an accident, so an arc with extreme consequences may remain desirable if the probability of an accident is low. Because the impacts on the population and nature have different units, a single arc 'cost' is derived by weighting the standardised arc 'costs'. Using a deterministic SP algorithm, (static) routes are determined for the range of weightings, implying different prioritisation, with the optimal route being that which minimises the sum of the 'normalised' impedances. HM routing problems are probably not a relevant application of SUMDP because the uncertainty is expressed in terms of the probability of an incident on a route. The revealing of uncertainty during a trip is in terms of there being, or not being, an incident while travelling along an arc. If there is an incident, the trip ends, but if there is not an incident, the optimal route, whether it is static or dynamic, remains optimal.

A major difference in HM problems is the technique used to quantify 'risk', and hence how the objective is defined (Erkut & Verter, 1998). The standard model assumes deterministic incident probabilities and consequences, and the objective minimises some combination of these as a single objective. Multi-attribute models have also been investigated e.g., an objective of minimising the total incident probability and population exposure. Erkut & Verter (1998) observed that the risk (defined as probability x consequence) for a particular route through a network is dependent on the risk associated with each link, and that this complication is simplified by assuming that the probabilities are so small that a suitable approximation is to minimise the sum of the risks on each arc, which can be achieved by finding the shortest route.

There is a body of literature which addresses the behaviour of travellers, and interaction between them e.g., Iida et. al. (1992), Abdel-aty et. al. (1997). Jansson & Ridderstolpe (1992) consider the impact on ridership distributions when travellers select amongst various routes to reach the same destination under various combinations of behavioural and operational assumptions. Chen & Hseuh (1998) consider the dynamic user-optimal departure time/route choice problem, though it is discussed in the context of a group of individuals selecting a route which minimises their individual travel times and adapting it. While SUMDP is not envisaged as a tool to address these issues, one could imagine it being embedded in these methods so that the behaviour of individuals with different preferences could be analysed.

While these approaches handle the dynamic aspect of the problem, they are unable to handle non-linear terminal value functions, which are a natural consideration when a DM faces uncertainty. Interestingly, in their conclusions, Psaraftis & Tsitsiklis (1993) note that extending their model to handle a time window with penalties for early and/or late arrival (i.e., a non-linear value function) would require the inclusion of the time dimension into the state space, though they do not elaborate. The extension they refer to is the essential feature of the SUMDP approach described in this thesis.

10.3 SUMDP for SRC problems

In this section, the application of SUMDP to the SRC problem is presented and illustrated using the example discussed in the previous section. Consider an acyclic network where each node (i) is connected only to its successor node (i+1), and multiple arcs connect each pair of nodes. (This restrictive assumption is relaxed later in the section). If t_i is the time taken from i to i+1, the total time taken to reach I is:

$$T = \sum_{i=1}^{I-1} t_i \tag{10.6}$$

As shown earlier, with a non-linear utility function and stochastic arc times, the objective can be stated as:

$$\max \mathrm{E}[U(T)] = \max \mathrm{E}\left[U\left(\sum_{i=1}^{I-1} t_i\right)\right] \tag{10.7}$$

This objective is non-separable i.e., for a 3 node network and route (a,b):

$$\mathrm{E}[U(t_a + t_b)] \neq \mathrm{E}[U(t_a)] + \mathrm{E}[U(t_b)] \tag{10.8}$$

The expected utility depends on the travel time taken over all stages, and cannot be calculated by adding the expected utility of each individual travel time, as noted in several articles when considering non-linear utility functions (Bard & Bennett 1991, Loui 1983, Murthy & Sarkar 1997). Therefore, the objective is non-separable, invalidating the use of a recursive relation which is independent of the outcomes of decisions taken at other nodes of the network. This invalidates the principle of optimality underlying **SRC1**, and presumably motivated the search for alternative solution approaches such as the complete enumeration formulation (**SRC2**).

In order to overcome the problem of non-separability, another state variable is defined. In the reservoir management problem, the variable was the accumulated profit up to the beginning of the period. In the SRC problem, the variable, T_i, is the accumulated time to reach i. Because the network is directed and acyclic, the node precedence relationships are known, so the time taken to reach node i is simply the sum of the arc lengths from the origin to i. This is equivalent to the wealth at the beginning of a particular period being the sum of the profits earned in all previous periods; the nodes in the SRC problem are just stages in a SDP.

The accumulated time taken to reach node i from the origin is therefore:

$$T_i = \sum_{\gamma=1}^{i-1} t_\gamma \tag{10.9}$$

The accumulated time to reach node i+1 is:

$$T_{i+1} = T_i + t_i \tag{10.10}$$

Thus, the total travel time to reach I is:

$$T_I = T_{i-1} + t_i = \sum_{i=1}^{I-1} t_i \tag{10.11}$$

which is the argument of the utility function described earlier. However, T_I is not dependent on all the decisions made prior to arriving at node I, but only on the value of T_{I-1} and the arc(s) selected at I-1 (which in turn affect the value of t_I). Similarly, T_{I-1} is dependent on the accumulated time and arc selection(s) made at node I-2, and so on. Thus, and more generally, if $M(i)$ is the set of nodes preceding node i, then T_i only depends on the value of $T_m \;\; m \in M(i)$ and the length of the arcs connecting m to i. As with the wealth in SUMDP model described for the reservoir planning problem, the initial travel time at the origin node (first stage) is assumed to be zero ($T_1 = 0$).

The problem can now be reformulated as a standard stochastic dynamic program with a 2-dimensional state space: the accumulated 'travel' time, T_i, and the node (or stage), i. The terminal value function, $f(I, T_I)$, is determined by evaluating the utility over a range of relevant travel times to the node I. It is possible to have multiple origin and destination nodes; they are equivalent to states of the first and last stages, respectively. Assuming there is a single destination and origin, the terminal value function is defined as $f(I,T_I)=U(T_I)$ and the stochastic dynamic programming recursion is:

SRC3
$$f(i,T_i)=\min_{j,k|j\in J(i),k\in K(i,j)}\left(\mathrm{E}\left[f(j,T_j)\right]\right) \qquad \forall i\neq I$$

subject to
$$T_j = T_i + t_k$$

$$T_1 = 0 \tag{10.12}$$

where t_k and $K(i,j)$ are defined as in the previous section. The decision to be made at each node is which arc to travel on, and this implies what the successor node will be, as an arc can only be associated with a single (i,j) pair. **SRC3** is solved using discrete dynamic programming. While the nodes and arcs are naturally discrete, the accumulated time is not, so for each node, T_i is discretised across the range of possible arrival times at i. The bounds on T_i can be calculated by passing through the network and calculating the minimum and maximum times each node can be arrived at given the bounds on T_i for predecessor nodes and relevant arc length distributions. There is clearly potential to refine this aspect of the solution technique, both in terms of intelligent discretisation of the T_i variables and the application of arc reduction techniques.

Starting from stage I-1 and working backwards according to the node precedence relationships, $f(i,T_i)$ is evaluated for each node and the optimal decision from each stage stored. With non-negative arc lengths, the range of T_i will increase for i further away from the origin, and T_I will have the longest range of arrival times. Negative arc lengths can be handled by the technique and require no special attention; they are equivalent to negative profits in the reservoir management problem described earlier.

The formulation in **SRC3** does not require that the utility function take a particular form. The form of utility function is dependent on the type of problem and decision making environment. For example, a 'decreasing deadline' utility function may be relevant for R&D project evaluation (Bard and Bennett, 1991), whereas a concave or other non-linear function may be relevant for a route choice problem where a DM specifies an attitude towards travel time and/or distance and/or cost. The formulation does not require a specific form of arc length uncertainty, either, though it will impact on the ease with which T_I is discretised. For example, if the distributions have an upper bound of $+\infty$ then a heuristically set upper bound will be necessary. Depending on the form of the uncertainty and utility functions, algorithmic modifications should be able to be developed to reduce the time required to determine optimal arcs, particularly when there are multiple arcs linking two nodes.

10.3.1 Example 1

Consider again the example network, arc data, and decreasing deadline utility function introduced earlier. Let $T_i^{\min}$ and $T_i^{\max}$ be the lower and upper bounds on the accumulated completion time at node i. The range of completion times for tasks 1 and 2 is 1-4 weeks, so the bounds on the arrival time at node B are $T_B^{\min}=1$ and $T_B^{\max}=4$ (because $T_A^{\min}=T_A^{\max}=0$). The maximum and minimum travel times of arcs 3 and 4, the two alternatives from node B, are 4 and 8 weeks, respectively. Thus $T_C^{\min}=T_B^{\min}+4=5$ hours, and $T_C^{\max}=T_B^{\max}+5=5$ hours.

Using the decreasing deadline utility function illustrated in Figure 10.2, the terminal value function is defined over the range of T_C as $f(C,T_C)=U(T_C)$. Now consider the task to select upon 'arrival' at node B. The expected values of $f(C,T_C)$ for the range of possible values of T_B and the two possible tasks, are plotted in Figure 10.3. The optimal task(s) could be either arc, depending on the length of time taken to arrive at B. For $T_B=1$ and $T_B=2$, arc 3 maximises expected utility with values of 75 and 66.8, respectively. For $T_B=3$ and $T_B=4$, arc 4 maximises expected utility with values of 56.2 and 42.6, respectively.

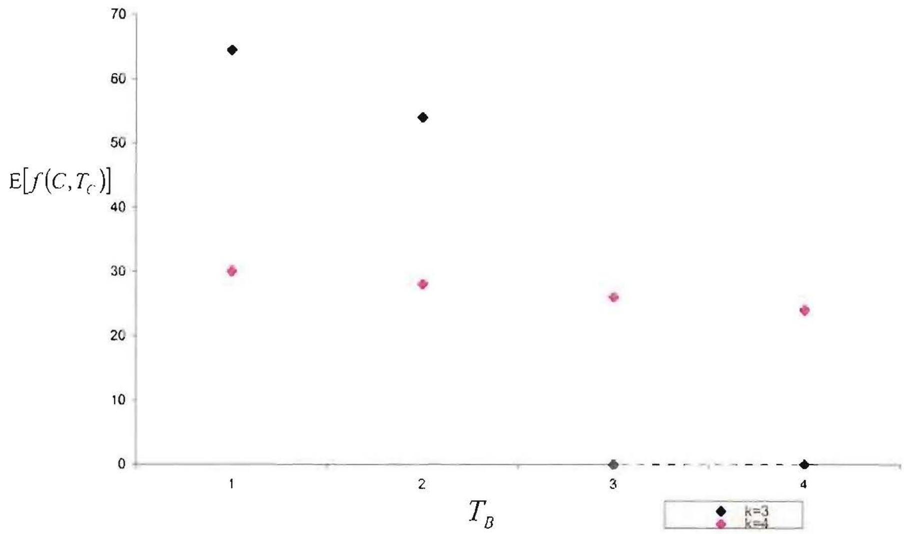


Figure 10.3: $E[f(C,T_C)]$ **values for arcs 3 and 4**

From A, the optimal task is that which maximises $E[f(B,T_B)]$. Task 1 has a duration of either 1 or 3 hours, so $E[f(B,T_B)]$=.4(56.2)+.6(75)=67.5. The corresponding value for arc 2 is $E[f(B,T_B)]$=57.1, so the optimal task from A is task 1. Task 1 only has two possible outcomes, though, so the optimal route can be stated as follows: select task 1 and if its duration is 1 week, then select task 3, otherwise select task 4 (see Figure 10.4).

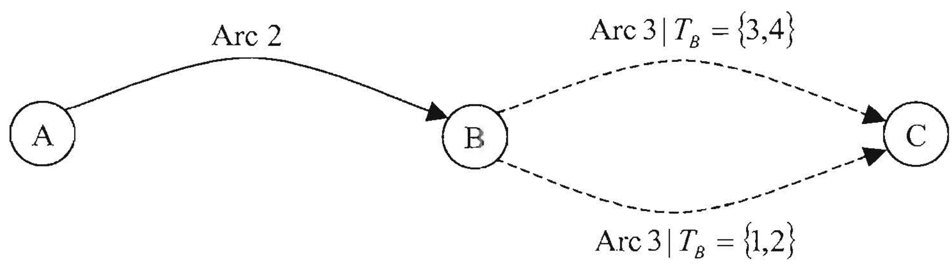


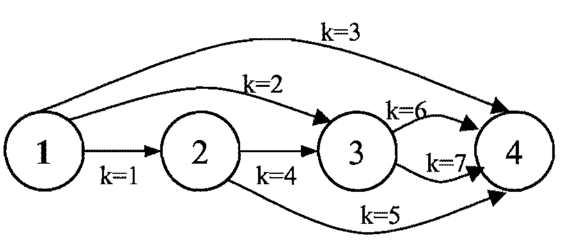
Figure 10.4: Solution to SRC3

This dynamic solution allows the DM to select the task at B given the realisation of uncertainty associated with the decision made at A. Moreover, the solution to even this simple problem involves the potential use of both tasks 3 and 4 at B, which would be impossible in a static solution.

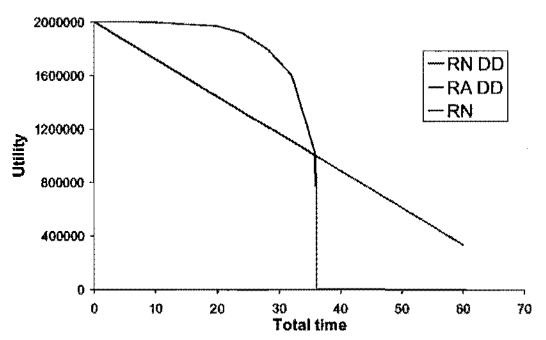
The example problem has been solved using three formulations (**SRC1**, **SRC2**, and **SRC3**), each with a different way of handling DM preferences. The optimal routes for **SRC2** and **SRC3** would be expected to be more sensitive to the variability of T_C, and particularly when $T_C > d$, at which point the benefit from completing the project is zero. This is certainly the case. The optimal route for **SRC1** has a lower expected completion time of 8.8 weeks compared to 9.1 (**SRC2**) and 9.08 (**SRC3**) weeks, and a lower range of completion times of 2 weeks compared to 5 for **SRC2** and **SRC3**. The difference between the utility maximising static (**SRC2**) and dynamic (**SRC3**) solutions is highlighted by an increase in the probability of meeting the deadline; $\Pr(T_C \le d)$ equals 0.27 for the optimal route to **SRC1** compared to 0.40 (**SRC2**) and 0.64 (**SRC3**). These are what would be expected for a risk averse DM – a distribution of completion times with a lower range and higher mean is preferred to a distribution with a lower mean and higher range (more variability).

10.3.2 Example 2

Consider a (small) SRC problem which involves finding a utility maximising route through the network illustrated Figure 10.5a. In this network, the DM has the ability to 'skip' nodes. The arc lengths are normally distributed as follows: arcs 1, 4, and 6 are $N \sim (11,2)$; arcs 2 and 5 are $N \sim (21,1)$; arc 3 is $N \sim (30,10)$, and arc 7 is $N \sim (12,1)$.



(a) Network



(b) Utility functions

Figure 10.5: SRC example

Three utility functions were defined over the range of completion times (Figure 10.5b). The base case is when the DM is risk neutral (RN), which corresponds to the negatively sloped dashed line. The other two utility functions reflect the case where there is a limit on the completion time, and this is reflected by a 'decreasing deadline' utility function described earlier. The curves differ in the modelling of preferences towards T_N when $T_N < d$, where d=36. The 'RNDD' utility curve is linear for $T_N < d$, while the 'RADD' utility curve is concave.

Discrete dynamic programming is again used to find a utility maximising route through the network. The solutions are summarised in Table 10.2. The RN solution (arc 3) takes no account of the variability of the arc times, so the (combination of) arcs with the lowest mean travel time is preferred. The RNDD solution involves using arc 2 to reach node 3, then either arc 6 or 7, depending on the time of arrival at node 3 i.e., the length of arc 2. The RADD solution takes a more conservative decision at node 1, moving to node 2 via arc 1. If the realisation of t_1 is less than 15.4, arc 5 is selected to move to the destination node. Arc 1 is $N \sim (11,2)$, so the route 1-4 is most likely to occur. However, if t_1>15.4, arc 4 is selected to reach node 3, at which time another set of conditional arc selections is described. The clear difference between the RADD and RNDD policies is that risk aversion results in a more flexible strategy which is more sensitive to the distributions of the arc lengths.

	Node 1	**Node 2**	**Node 3**
RN	Arc 3	-	-
RNDD	Arc 2	-	Arc 6: $T_3 < 19$ $21 < T_3 < 22.4$ $T_3 > 22.8$ Arc 7: $19 < T_3 < 21$ $22.4 < T_3 < 22.8$
RADD	Arc 1	Arc 5: $T_2 < 15.4$ Arc 4: $T_2 > 15.4$	Arc 6: $T_3 < 18.9$ $21.2 < T_3 < 22.4$ $T_3 > 22.8$ Arc 7: $18.9 < T_3 < 21.2$ $22.4 < T_3 < 22.8$

Table 10.2: SRC solution

Consider the decision process at node 3 for the RNDD and RADD cases. For low (good) T_3 where there is essentially no chance of T_4>d, the means and variances of arcs 6 and 7 are such that arc 6 has a higher expected utility than arc 7. Let μ_k denote the mean length of arc k. Because the means and variances of arcs 6 and 7 are similar, for T_3 near d- μ_k, there are ranges of T_3 for which each arc is preferred. However, arc 6 has a lower mean and higher variance, so for large values of T_3 there will be a higher probability that $T_4<d$, and hence an expected utility greater than that for arc 7. Arc 6 is therefore preferred for both RADD and RNDD when T_3 is large.

10.4 Extensions to the formulation

This section briefly discusses some extensions to, and other application if, SUMDP to stochastic route choice problems.

10.4.1 Generic extensions

Three formulations have been presented for SRC problems with uncertain arc travel times. The fundamental difference between the three formulations is that in **SRC1** the objective is to minimise the expected travel time and in **SRC2/SRC3** the objective is to maximise the utility of the expected travel time. A natural variation of **SRC3** is to consider travel cost instead of travel time. The formulation would remain essentially the same, but instead of accumulating time, cost would be accumulated, with C_i being the cost incurred to reach node i and c_k being cost distribution if arc k is used to travel between two nodes. The uncertainty could be a function of a number of factors, such as the travel time, distance of the route, uncertainty in the traffic density, and road conditions. The terminal value function would be defined over the range of costs, so $f(I,C_I)=U(C_I)$.

An interesting, yet more computationally intensive formulation would involve the utility function being based on both time and cost i.e.,

SRC4
$$f(i,T_i,C_i)=\max_{j,k|j\in J(i),k\in K(i,j)} \mathrm{E}\left[f(j,T_j,C_j)\right]$$

subject to
$$C_j=C_i+c_k$$

$$T_j=T_i+t_k$$

$$T_1=0$$

$$C_1 = 0 \tag{10.13}$$

This is a SDP with a 3-dimensional state-space and could be used to model a SRC problem where there are alternatives with different fixed and variable costs, as well as different time distributions.[9]

Consider now an extension to the SRC where travel times (arc lengths) are uncertain and dependent on the time of arrival at a node. The 'accumulated time' state provides the information to handle time-dependent travel times. In, the previous formulation, T_i refers to the accumulated travel time upon reaching node i and is implicitly independent of real time i.e., T_i is the 'time since departure' and the departure time is either assumed (and fixed), or irrelevant. Examples of time-dependent travel times are the peak and off-peak traffic flows on urban road networks. A simple stepped travel-time function could be used to reflect these time dependencies. Travel time distributions would be associated with each congestion segment. If time-dependent travel times are being considered, then the network must be anchored to real time in some way. One way to achieve this is to explicitly define a departure time from node 1, r_1, which is used as a reference point to calculate the 'real' arrival times at a particular node. If R_i is defined as the 'real' arrival time at node i, then $R_i = f(T_i, r_1)$ and the state transition becomes $T_j = T_i + t_k(R_i)$. A formulation for this problem is

SRC5 $$f(i,T_i) = \min_{j,k|j\in J(i),k\in K(i,j)} \left(\mathrm{E}\lfloor f(j,T_j)\rfloor\right) \qquad \forall i \neq I$$

subject to $$T_j = T_i + t_k(R_i)$$

$$R_i = f(T_i, r_1)$$

$$T_1 = 0 \tag{10.14}$$

The utility function is still defined over the range of possible T_I values, which will correspond to a range of real-time values (R_I). If the function $f(T_i, r_1)$ is additive then $U(T_I)$ and $U(R_I)$ will be linear transformations of each other. Note that using $U(T_I)$

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[9] For example, consider travelling between two cities by car and/or plane and/or train.

or $U(R_I)$ implies that the DM has no preferences regarding the real-time departure time, r_1. When arc travel times are independent of node arrival time, this assumption is reasonable, though not necessarily realistic. Note, though, that the sensitivity of the optimal route to different departure times could be examined by adjusting r_1.

An alternative to time-dependent travel times is to have correlated travel times which reflects the fact that observing a high traffic density along one route is likely to mean that the next route has a high density too. It could also be a way of representing the effect of getting a "run" on the lights or striking a bad "run" where red lights (and hence slow travel times) are repeatedly experienced. This relationship could be modelled by a Markov process where the discrete states are the different traffic densities. A 2-state transition process with high same-state transition probabilities could be used to model this case. A formulation for this problem would be as follows. The terminal value function is defined as

$$f(I,T_I,M_I)=U(T_I) \tag{10.15}$$

The state space for this problem comprises the nodes (i), the accumulated travel time (T_i), and the state of the traffic (M_i). This problem can be formulated as a SDP as follows:

SRC6 $$f(i,T_i,M_i)=\min_{j,k|j\in J(i),k\in K(i,j)}\left(\mathrm{E}\left[f(j,T_j,M_j)\,|\,M_i\right]\right) \qquad \forall i\neq I$$

subject to $$T_j=T_i+t_k k(M_i)$$

$$T_1=0 \tag{10.16}$$

where M_i is the Markov state of the uncertainty process (affecting travel time) at node i where there are some number of discrete states (N_m). The transition probabilities would exacerbate the differences between different types of routes and transport mediums. For example, in a network comprised of shorter/congested urban roads and longer/less congested ringroads, there would be a high probability of the urban road remaining congested, while on the ringroad, there would be a high probability that the congestion remained low.

In **SRC5** the travel times are dependent on the time of arrival, but independent between arcs. In **SRC6**, the travel times are dependent on the type of 'congestion' observed along the previous arc, but are independent of the actual time of day. Combining the two approaches is a natural extension to both models.

SRC7 $$f(i,T_i,M_i) = \min_{j,k|j\in J(i),k\in K(i,j)} \left(\mathrm{E}\left[f(j,T_j,M_j)\,|\,M_i\right]\right) \qquad \forall i \neq I$$

subject to $$T_j = T_i + t_k(T_i', M_i)$$

$$T_i' = T_i + d_1$$

$$T_1 = 0 \qquad (10.17)$$

Previous formulations assume the utility function only applies to the destination node. It is easy to construct scenarios where the DM has multiple preference functions, and/or multiple trips. It would be useful to extend SUMDP to handle the case where the DM has a preference towards some other trip component other than the arrival time at the destination node e.g., the arrival time at an intermediary node. Consider the situation depicted in Figure 10.6. The DM must travel from O1-D1 where some activity is performed. Upon completion of the activity, the DM takes another trip (O2-D2), possibly to return to the original origin. The DM may have preferences towards the time taken for trip 1 (arrival time at the destination), the time available for the activity, and the time taken for trip 2. The activity completion time may be deterministic or stochastic, and may be able to be influenced if the DM takes some action, presumably at some cost. What are the optimal routes for trip 1 and 2, and optimal 'activity action(s)', given these preferences?

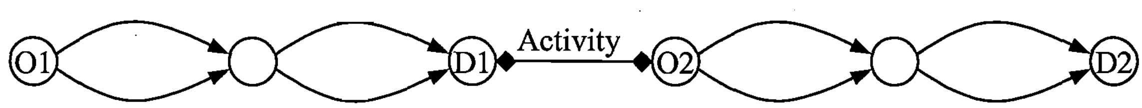


Figure 10.6: A trip-activity-trip scenario

Another variation relates to the time the DM departs from the origin node. In the above formulations, the departure time is assumed to be fixed and known in advance. This is a reasonable assumption for those applications for which the departure time has little direct impact on the DM's utility or on the nature of the uncertainty modelled in the network (e.g. project scheduling). When the departure time does affect the form of the problem (e.g., time-dependent travel times) then an obvious extension to these models is to relax the assumption that the departure time is fixed. If r_1 is to be considered as a variable, then the DM's preferences towards r_1 become important. If the DM is indifferent between values of r_1, then, depending on the form of $U(T_I)$, it may be optimal to leave at an unreasonably early time in order to arrive at some destination prior to some deadline. Were $U(T_I)$ to be monotonically decreasing in T_I there is no incentive to leave 'late' so the early departure will likely result in an early arrival and therefore maximise utility. If $U(T_I)$ had positive and negative sloping regions then early arrival is less undesirable and an 'unreasonable' r_1 will not be utility maximising. These behavioural aspects of traveller behaviour are an area worthy of future development.

10.4.2 Project selection problems

The planning of projects/tasks can be represented by a network where the nodes represent projects and the arcs represent project completion times and precedence relationships. The standard analysis is to determine the distribution of the longest/shortest route, so there is no optimisation as such. When the DM can select projects/tasks (as in R&D project selection), then the problem can be modelled as a DP. The SUMDP approach can be applied to the stochastic project selection problem (also referred to as probabilistic PERT and PNET) by simply replacing the definition of the nodes and arcs. The literature and some extensions to the stochastic project scheduling problem are discussed below.

Dawood (1998) considers the project scheduling/selection problem where the duration of a particular activity (arc) is affected by a number of 'risk factors' which can have different probability distributions. He discusses the problem in the context of managing a construction project, so the risk factors include weather, soil types, and design changes. The variability of the project's duration is determined by performing Monte Carlo simulations where the random numbers, in conjunction with subjective weightings, are used to determine activity duration times. Using a correlation analysis, the risk factors which have the most impact on project duration (which were not dissimilar to those derived using a standard PERT approach) are identified. The only decision making process involves an adjustment of the weights for risk factors to reflect a management decision (e.g., reduce the impact of weather if the project is starting over summer). The simulations are re-run, and, not surprisingly, the variability in project duration time is reduced. While simplistic, the approach is interesting in that the impact of risk factors is explicitly considered.

Bard & Miller (1989) present a heuristic approach to the project selection problem when the DM also has a fixed budget which can be 'invested' in the projects to improve the distribution of expected completion time. A "near optimal" solution is found by a heuristic procedure which takes as inputs the results from a Monte Carlo simulation, where each simulation involves determining a static, utility maximising, optimal sequence of projects and investments given a particular realisation of route lengths. The objective of each realisation is to maximise expected utility, though this is only based upon time; they assume that there is no value in having any of the budget remaining upon reaching the destination node. They test their approach on some small sized problems, noting that their method is computationally intensive and that arc reduction methods would be a useful development. The solution is static, yet a dynamic solution would be a useful extension because one could imagine that having been selected, the duration of a given project would affect which project is selected once it is completed. This extension does not appear to have been addressed in the literature. Later, a formulation is presented, along with some further extensions.

The approach of Bard and Miller (1989) is similar to the reservoir management problem in that a 'stock' (the remaining budget/water) is carried through from one stage to another. By augmenting the problem discussed by Bard and Miller with an 'accumulated arc length' (or accumulated time) state, DM preferences towards terminal budget funds and total project time on project selection can be explicitly modelled. The optimal solution describes the project to be selected and the budget allocation that maximises the DM's expected terminal value upon completing the last project. In Bard & Miller (1989), the utility function is a function of the total time taken to complete the project, and the entire budget must be allocated. Here, though, the DM's utility can be a function of the remaining budget i.e.,

$$f(i,T_I,B_I)=U(T_I,B_I) \tag{10.18}$$

where

I final node in network.

T_I total time taken to reach node I.

B_I dollars available left upon reaching node I.

This is more realistic than Bard and Miller (1989) and requires no additional information because the budget level is a state variable. The objective to be maximised at each node is

SRC8 $$f(i,T_i,B_i) = \max_{j \in J(i), y_i} \mathrm{E}\lfloor f(j,T_j,B_j) \rfloor$$

subject to:

$$B_j = B_i - y_i$$

$$T_j = T_i + L_i(y_i)$$

$$b_1 = B^{init}$$

$$\underline{B_i} \le B_i \le \overline{B_i} \tag{10.19}$$

where

$f(i,T_i,B_i)$ expected terminal utility if at node i with an accumulated arc length of T_i and having B_i units remaining to be allocated.

i node number i=1,…,I.

$J(i)$ the set of feasible nodes (denoted by j) to travel to at node i.

B_i dollars available to allocate at node i.

T_i time spent to reach node i.

B^{init} initial budget allocation.

y_i dollars allocated at node i.

$L_i(j,y_i)$ the project completion time distribution given an investment of y_i and that the decision is to complete the project represented by the arc connecting i and j.

Upon arriving at node *i*, the expected utility of arriving at node *j* (via the project implied by the arc connecting *i* and *j*) is determined. If y_i=0 then the budget remains unchanged ($b_j = b_i$) and the range and mean of the possible project times is at its largest. Allocating y_i>0 reduces the range and probability of a long project time eventuating, though the remaining budget is also reduced. This problem can be solved by discretising the 'remaining budget' and 'accumulated project time' states.

A small problem was coded and solved and is briefly discussed here. It is similar to the problem described by Bard & Miller (1989) and involves finding the utility maximising path through the network illustrated in Figure 10.7, where the nodes correspond to projects with uncertain completion times. As described in the above formulation, a budget can be allocated to each project and this affects the distribution of the time taken to complete a project. The decision to be made at each node is how much to invest in the current project, given the time taken to complete the previous projects, and which project (node) will be completed following the completion of the current project. Note that this somewhat unintuitive state transition is due to the way in which the network has been described. It is equivalent to a network with 4 'decision' nodes with arcs between nodes corresponding to projects. The decision at each node would be which project to select, and how much to spend on it; decisions about future projects would not be considered until the 'current' project had been completed.

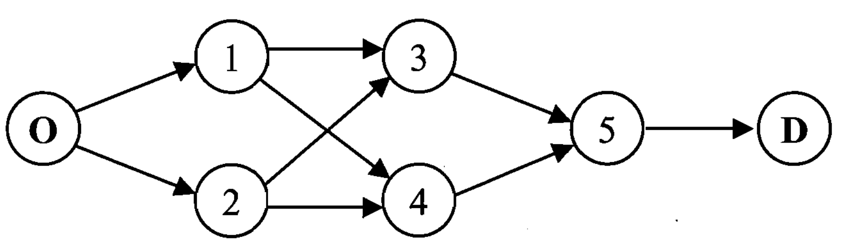


Figure 10.7: Network for project scheduling example

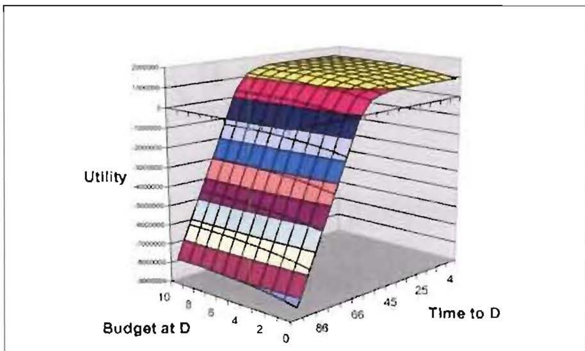
The length of each project was approximated using a Normal distribution. The project length data is detailed in Table 10.3. Also included in the table are Δ_1 and Δ_2, which are parameters used to improve the project length distributions when capital is allocated to them. If μ and σ denote the mean and standard deviation of a project length and *y* is allocated to a project, then the distribution of the project length was drawn from $N(\mu', \sigma')$ where $\mu' = \mu - y\Delta_1$ and $\sigma' = \sigma - y\Delta_2$. A total budget of 10 monetary units is available at the start of the planning process.

Project	Mean	Standard deviation	Δ_1	Δ_2
1	18	4	0.05	0.1
2	22	4	0.1	0.05
3	22	6	0.05	0.05
4	25	3	0.05	0.05
5	20	2	0.05	0.05

Table 10.3: Project length data

The utility function used was that shown in Figure 10.8. The solution is also shown in Figure 10.8, and involves the following sequence of projects: 2-3-5. Although the initial budget available was fixed at 10, the optimal project changes from 2 to 1 when the initial budget is less than or equal to 6. With a lower budget, the ability to reduce the variation in project time is reduced, so a more conservative strategy is taken for the first project so that the remaining budget can be better allocated to reduce the distributions of task 4 and/or 5.

While the project selections turned out to be static, the budget allocations are not (Figure 10.9). They have an intuitively pleasing form, being convex in both accumulated time and remaining budget. Thus, the amount spent on the project increases as the time spent on previous projects increases, and as the remaining budget increases.



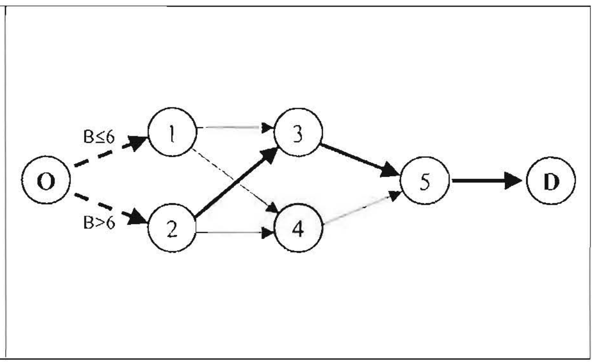


Figure 10.8: Utility function and solution

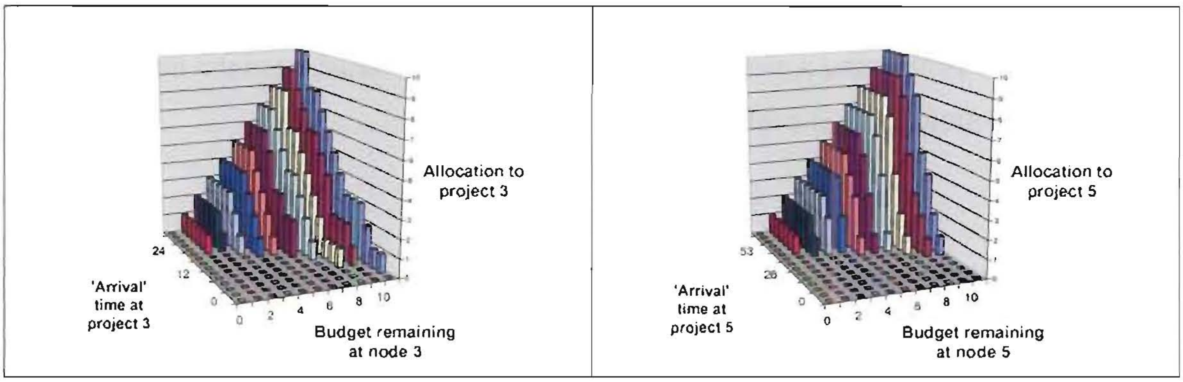


Figure 10.9: Dynamic budget allocations

For large networks, the SDP algorithm could be quite computationally intensive because at each node the optimal arc (project) for a relevant range of budget levels and accumulated project times would need to be determined. It may be possible to reduce the search by being able to show that a given project is optimal for some range of budget and/or accumulated project lengths, though this will depend on the form of the utility function and the nature of the state transition. The formulation stated above has the arc length distributions dependent on the budget level, so the dominance of one project over another for a given budget level may not be applicable for another budget level. Dominance relationships for a given budget level would presumably provide useful starting points for the search algorithm, though, and it may be that all that is required is that any additional feasible budget allocations are evaluated, rather than having to evaluate $f(i, T_i, B_i)$ for all budget allocations. This is an area for investigation and experimentation should a model be developed and tested.

A variation is to consider the case where there is some initial budget which has random contributions made at the completion (or prior to selection) of each project. The state transition for the budget state would then become:

$$B_j = B_i - y_i + c_i \tag{10.20}$$

where c_i is a random contribution towards the budget. If the distribution of c_i is independent of the level of B_i and T_i, then an intermediate expected utility surface can be created, as in the reservoir management problem, which is the convolution of the contribution distribution and the expected utility surface at node j.

Contributions could be a performance or reward measure, perhaps reflecting the desirability of completing a particular project and/or being based upon the time taken to complete the project. This could be modelled as

$$B_j = B_i - y_i + c_i\left(j, L_i\left(j, y_i\right)\right) \tag{10.21}$$

so the budget remaining upon reaching node j is the budget at node i, less the 'investment' in the project plus a contribution or return which is a function of the project and the expected time to complete the project. The concept of time-dependent travel times could be translated to wealth-dependent incomes. For example, consider a scenario where the decision to be made at each period is to invest some amount in a 'product' which has an uncertain return. Holding money also has a return, but this is dependent on the quantity of money held because a higher holding allows the DM access to a better rate.

10.4.3 A capacity expansion problem

SRC problems are often discussed in the telecommunications literature in the context of capacity expansion, though there are few approaches which consider stochasticity Laguna (1998). The decision to be made in each period is how much additional capacity to install given that the demand for capacity is uncertain. Consider the situation faced by a (telecommunications or transmission) firm that needs to determine the way in which it should manage its capacity for a single location over time given that demand for capacity is uncertain. This situation is represented in Figure 10.10, where the horizon is four periods and the capacity of the network is represented by the x variables.

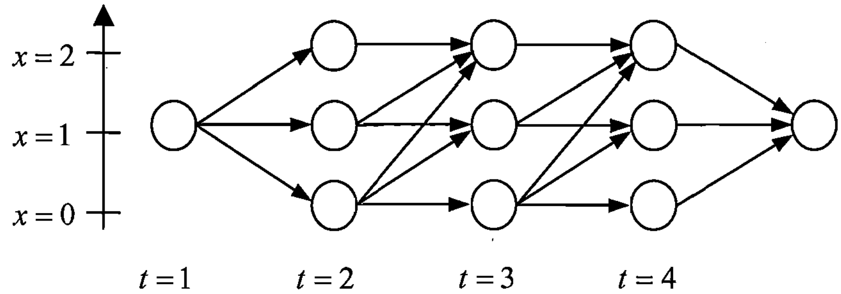


Figure 10.10: Illustrative capacity expansion network

During each period, some level of demand, d_t, is experienced, which is independent of the previous demand level. The distribution of demand is estimated with some known probability distribution $P(d_t)$. In a given period, the cost of not meeting demand, $\gamma(d_t - x_t - y_t)$, incurs a real (or perceived) cost. The variable y_t denotes the capacity installed at the beginning of period *t*, prior to the demand being experienced. Note that if $x_t + y_t \geq d_t$, no cost is incurred.

Increasing capacity also incurs a cost $c_t(y_t)$, where y_t is the number of units installed in period *t*. The cost function would usually involve a fixed cost and then linear or convex variable costs. The objective is to minimise the installation costs and 'unmet demand' costs, i.e.

$$z = \sum_{t=1}^{T} (c_t(y_t) + UDC_t) \tag{10.22}$$

where

$$UDC_t = \int_{x_t}^{\infty} P(d_t)\gamma(d_t - x_t - y_t)\partial t \tag{10.23}$$

Alternative forms of *z* could include a maintenance cost term, and an end-of horizon value of capacity term.

The decision to be made in each period is the number of units of capacity to install and the feasible (discrete) alternatives are represented by the arcs linked nodes. If the utility function were based on the cost at the end of the horizon, utility would be defined as $U(z) = U\left(\sum_{t=1}^{T} (c_t(y_t) + UDC_t)\right)$. As with the simple SUMDP model, the utility function is handled by defining another state variable, z_t, which is the accumulated installation and unmet demand cost at period *t* where the state transition equation is $z_{t+1} = z_t + c_t(y_t) + UDC_t$ and $z_1 = 0$. The state space of this problem therefore consists of the time period (*t*), the quantity of installed capacity at the beginning of period *t* (x_t), and the accumulated cost at the beginning of period *t* (z_t). The terminal value function can be defined as $f(t, x_T, z_T) = U(z_T)$. An alternative is to define $f(t,x_T,z_T)=U(x_T,z_T)$ to incorporate the firm's attitude to the capacity installed in T. The recursive relation can now be defined as

SRC9
$$f(t,x_t,z_t)=\max_{y_t \text{ feasible}} \mathrm{E}\left[f(t+1,x_{t+1},z_{t+1})\right]$$

subject to
$$x_{t+1}=x_t+y_t$$

$$z_{t+1}=z_t+c_t(y_t)+UDC_t$$

$$\underline{x}\le x_t \le \overline{x}$$

$$z_1=0 \tag{10.24}$$

Solving this problem would require that the state and decision variables be discretised, as indicated by the form of the network. Feasible extensions to this problem include the installation cost being dependent on the current level of installed capacity, and demand uncertainty being modelled as a Markov process (requiring the definition of another state variable).

This formulation assumes a simple representation of a complex problem, though there appears to be a lot of potential in developing a dynamic solution to this problem. For example, in the above model the decision variable is the quantity of capacity to install for a single component. The handling of multiple components is an obvious extension and this could be achieved by solving a more complex sub-problem (in Laguna 1998 the sub-problems are stochastic knapsack problems, which is itself yet another potential application of SUMDP).

10.5 Conclusions

There are many problems which have solutions corresponding to 'shortest' routes through an acyclic network. Stochasticity makes these problems more difficult to solve, and this is compounded if a DM's preferences toward terminal outcomes is considered. To the author's knowledge, there are no approaches presented in the literature for finding a dynamic utility maximising path through a stochastic network. Although the technique was illustrated using an extremely simple example, there appear to be a number of extensions to other stochastic planning problems that can be represented as acyclic networks where uncertainty is revealed over time.

Chapter 11

Conclusions

This chapter summarises the central ideas and results from applying SUMDP (an approach for maximising utility in a stochastic dynamic programming framework) to medium-term reservoir management in deregulated and regulated electricity markets. SUMDP was also explored as a solution approach for stochastic route choice problems.

11.1 SUMDP

Stochastic dynamic programming is a popular technique for modelling finite horizon stochastic sequential decision problems; one such problem is reservoir management. A frequently used objective for SDP models is to maximise the expected value of the outcomes of decisions. This implies a risk neutral attitude to outcomes which is not always appropriate, and often just a convenient simplification due to the difficulty of handling risk aversion.

A utility function can be used to reflect risk aversion, but maximising utility is an invalid objective for a standard SDP formulation, and computationally intractable if solved via a decision tree. In SUMDP was introduced as a method for evaluating a decision tree which is more efficient that complete enumeration due to the stage-wise decomposition. This was achieved by defining an auxiliary variable as the 'accumulated returns' and assuming that the DM's preferences are defined over the sum of the returns ('accumulated returns') at the end of the planning horizon. As a result, a SUMDP formulation allows the maximisation of the utility finally realised in T without requiring that all decisions up to T be considered simultaneously.

The practical and theoretical applications of such a technique are broad due to the relative simplicity of the conditions required to extend a SDP formulation to handle the 'utility maximising' formulation. A drawback, though, is that the auxiliary variable increases the number of states evaluated at each stage. Methods for reducing the impact of this are dependent on the characteristics of the problem, and should be investigated for any application.

11.2 SUMDP and medium-term reservoir management

Medium-term reservoir management is a classic planning problem to which SDP has been applied, particularly in New Zealand where there is a high dependence on hydro generation. Risk has been identified as being of prime importance for reservoir management problems, and reservoir management in New Zealand is no exception. The environment in which risk is considered has also changed with the reservoirs managed in deregulated electricity markets rather than by regulated and/or Government-owned entities. In these two contexts, there are few published approaches that incorporate the DM's attitudes to outcomes at the end of the planning horizon, and few approaches that consider reservoir management in a deregulated electricity market where there are few firms with market power.

Chapters 4-8 discussed the application of SUMDP in the context of reservoir management in New Zealand, assuming regulated and deregulated electricity markets. The form of the utility functions used here implied a trade-off between storage and 'wealth', as opposed to only being risk averse towards wealth, so the impact of these functions of both components is important. In the regulated case, the benefit from release was the cost of meeting demand not met from reservoir release. Both these return functions use simplified representations of the New Zealand electricity system, and do not take into account transmission effects, or issues such as contract trading, the effect of spot market operation on contract prices/revenue, and the threat of new entry on market behaviour.

Experimental results for both cases clearly showed that as the risk aversion towards end-of-horizon wealth increased:

- the mean and variability of end-of-horizon wealth decreased.
- the mean and variability of end-of-horizon storage increased.
- Storage trajectories were higher on average and releases lower on average, storage being a mechanism for hedging against the impacts of future inflow uncertainty.
- Spot prices tended to increase due to lower release levels.

For the case of a dominant hydro firm which was contracted (and in this case contracted for more than its average output), the experimental results showed that as the contract level was decreased towards expected output:

- Mean end-of-horizon wealth decreased slightly, though the variance of end-of-horizon wealth decreased substantially.
- The spot price increased, reflecting the decreased incentive for the firm to push spot prices lower.
- Mean and standard deviation of end-of-horizon storage remained relatively constant.

The magnitude of the effects of risk aversion on the end-oh-horizon wealth and storage distributions was greater than those resulting from altering the contract level. However, decreasing the contract level resulted in end-of-horizon wealth distributions with extremely low variability, reflecting the ability of the hydro firm to reduce the frequency of situations which result in relatively extreme difference payments and spot revenues, and hence increase the variability of end-of-horizon wealth outcomes.

Throughout these analyses, a single reservoir model has been used. A two-reservoir model would likely produce more accurate results for the New Zealand system given the geographical layout of the hydro generation capacity and nature of the main transmission constraints. However, the resulting SDP formulation would have 3 state variables and would hence be computationally demanding.

11.3 SUMDP and stochastic route choice

It was observed that SUMDP can be extended to a range of problems where the accumulated return is a function of a subset of decisions in previous stages, as long as a stage is not returned to. Problems with a temporal aspect naturally have this property, though it can also be found in other problems which can be represented as a (stochastic) directed acyclic network.

The application of SUMDP to stochastic route choice problems was illustrated using several examples. They illustrated how a utility function defined over an attribute (e.g., time, cost) at the terminal node can result in a non-static solution at each node. This non-static property was clearly evident in the project selection and budget allocation example taken from the literature. While the project selection decisions for the example were independent of the total project length, the budget allocation decisions were non-static and a function of the total project length. This attribute of the optimal decision space was an enhancement to that provided in the published example.

11.4 Future research

The applications of SUMDP discussed here for reservoir management in New Zealand have defined wealth as the weekly return from release. In this context, a more realistic weekly model of the New Zealand electricity market would be desirable, especially now that information on bidding behaviour and contracts is becoming available. Incorporating transmission, ancillary services, and demand-side participation would also lead to a more accurate model. Another obvious extension is to consider correlated inflows, especially given that inflow uncertainty is the key driver of variability in outcomes. While this would increase the state-space, it would be possible to limit the increase in the computational requirements to a linear rather than exponential increase.

The ability to model aversion towards outcomes can be utilised in ways other than modelling risk aversion towards profit outcomes. For example: utility could be defined on the return which would be received by a potential new entrant. Briefly, if the firm knew the cost structure of a potential entrant, it would be possible to model the entrant's accumulated profit, and define a utility curve over that range so as to ensure entry to the market did not appear to be viable given the risks and returns available. A variant of this would be to model the performance of a price-taking competitor that owns hydro and/or thermal plant and faces uncertain prices and/or inflows. Another example would be to model a fuel stockpile used by a thermal plant that could be managed given uncertainties about alternative fuel prices/availability and generation requirements.

While it is possible to devise numerous potential applications, a key issue to be considered is how to integrate an approach such as SUMDP with the strategic planning process of organisations involved in managing hydro reservoirs and other assets for which a SDP and utility maximisation has value.

SUMDP has been used to investigate the impacts of risk aversion on storage in the medium-term. However, a longer-term approach addressing this aspect along with the interactions with spot prices, contracts, and demand is warranted. The forthcoming Ph.D. thesis of Batstone (2002) addresses some of these issues.

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Appendix 1

Multi-attribute utility functions

One approach for handling multiple attributes is to define an additive utility function (see for example, Keeney, 1970; Richard, 1975). For the reservoir management problem described earlier, this would have the form

$$U\left(w^{T+1}, s^{T+1}\right) = k_w u_w\left(w^{T+1}\right) + k_w u_s\left(s^{T+1}\right) \tag{A1.1}$$

Scalars k_w and k_s can be used to weight the utility functions. A function with this form implies that W^{T+1} and S^{T+1} are mutually utility independent; the utility associated with each consequence is independent of the value of the other consequence. For example, W^{T+1} would be utility independent of S^{T+1} if the (conditional) utility function for $w^{T+1} \in W^{T+1}$ given $s_1^{T+1}, s_2^{T+1} \in S^{T+1}$ does not depend on the values of s_1^{T+1} and s_2^{T+1}. As an aside, an additive utility function implies that W^{T+1} and S^{T+1} are utility independent, but not vice versa (Keeney and Raiffa, 1976). For some given arbitrary s_1^{T+1} and w_1^{T+1}, $w^{T+1} \in W^{T+1}$, and $s^{T+1} \in S^{T+1}$, additive independence implies that $pu\left(w^{T+1}, s^{T+1}\right) + (1-p)u\left(w_1^{T+1}, s_1^{T+1}\right) = pu\left(w^{T+1}, s_1^{T+1}\right) + (1-p)u\left(w_1^{T+1}, s^{T+1}\right)$. In terms of actually assessing the DM's utility function, additive independence reduces the difficulty involved in assessing $U\left(w^{T+1}, s^{T+1}\right)$ because only two conditional utility functions need to be derived in order to define utility over $W^{T+1} \times S^{T+1}$ (Keeney and Raiffa, 1976).

Additively independent utility functions can be difficult to justify, though, because of the assumption that the DM's utility is not affected by the value of the other attribute. A more general case is a multi-linear utility function, which has the form

$$U\left(w^{T+1}, s^{T+1}\right) = k_w u_w\left(w^{T+1}\right) + k_s u_s\left(s^{T+1}\right) + k_{ws} k_w k_s u_s\left(s^{T+1}\right) u_s\left(s^{T+1}\right) \tag{A1.2}$$

with $k_w, k_s > 0$, and W^{T+1} and S^{T+1} utility independent. The latter term is a scaling mechanism on the conditional utility functions, so the interaction between the utilities for W^{T+1} and S^{T+1} are accounted for. If the scaling factor in the last term is non-zero, this function can be expressed in multiplicative form

$$U\left(w^{T+1}, s^{T+1}\right) = u\left(w^{T+1}, s_1^{T+1}\right) u\left(w_1^{T+1}, s^{T+1}\right) \tag{A1.3}$$

using the conditional utility functions for W^{T+1} and S^{T+1}.

To assume that W^{T+1} is utility independent implies that the DM's utility for an increase in accumulated return is independent of the system state. If this implication is less plausible, a utility function can be defined to reflect S^{T+1} being utility independent of W^{T+1}, but not vice versa. Keeney and Raiffa (1976) state that, for $w_1^{T+1} \in W^{T+1}$, the utility function which handles this situation is

$$U\left(w^{T+1}, s^{T+1}\right) = c_1\left(w^{T+1}\right) + c_2\left(w^{T+1}\right) u\left(w_1^{T+1}, s^{T+1}\right) \tag{A1.4}$$

for some arbitrary $w_1^{T+1} \in W^{T+1}$, $c_2\left(w^{T+1}\right) > 0$. They go on to state how $c_1\left(w^{T+1}\right)$ and $c_2\left(w^{T+1}\right)$ can be derived from conditional utility functions (which have one of the attributes fixed) and/or isopreference curves (where the value of utility is constant). If neither S^{T+1} or W^{T+1} are utility independent, this definition of $U\left(w^{T+1}, s^{T+1}\right)$ may provide an adequate approximation to the DM's utility function (Keeney and Raiffa; 1976).

Richard (1975) gives conditions for comparing the risk aversion of two multi-attribute utility functions $U(w,s)$ and $V(w,s)$. Consider a DM's preferences for the following equally probable outcomes: $A=((w_0,s_0),(w_0,s_0))$ and $B=((w_1,s_0),(w_0,s_1))$, where $w_1 > w_0$ and $s_1 > s_0$. Strict risk aversion and risk seeking attitudes are implied for the above risk aversion and seeking attitudes if $A \sim B$ never occurs. Assuming that $U(w,s)$ is twice continuously differentiable in s and w and the first derivatives are positive, Richard (1975) states that

- iff $\partial U(w,s)/\partial w\partial s \leq 0$ then the DM is risk averse;
- iff $\partial U(w,s)/\partial w\partial s = 0$ then the DM is risk neutral; and
- iff $\partial U(w,s)/\partial w\partial s \geq 0$ then the DM is risk seeking.

Risk aversion receives the most attention in the literature because trading off returns and the risks inherent in attaining those returns is a characteristic of many decision problems, particularly in the financial area. Consequently, following the initial theories of utility functions and risk aversion, measures of risk aversion were developed. For example, the Pratt-Arrow measure of absolute risk aversion at a particular value of w is the ratio of $\partial U(w)/\partial^2 w$ to $\partial U(w)/\partial^2 w$. Recently, research has addressed the concept of representing (and measuring and assessing) attitudes to risk and return separately (e.g., Dyer and Sarin, 1982; Bell, 1995; Weber and Milliman, 1997).

Clearly, if $U(w,s)$ is multi-attribute risk seeking, then $V(w,s)$ has greater multi-attribute risk aversion if it is risk neutral or risk averse. The more interesting case is when both functions are multi-attribute risk averse. In that case, $V(w,s)$ has greater multi-attribute risk aversion than $U(w,s)$ if the multi-attribute risk premium (MRP) for $V(w,s)$ is greater or equal to that for $U(w,s)$ for the same uncertain consequences. Here, the MRP is a pair of values (w',s') which results in the same expected utility as the utility of the expected values. For the single attribute case, the risk premium (RP) is calculated as the difference between $\mathrm{E}(\tilde{w})$ and $\hat{w}$, which is known as certainty equivalent. For uncertain $\tilde{w}$, the certainty equivalent is the amount which results in $U(\hat{w}) = E[u(\tilde{w})]$. The RP is the difference between $\tilde{w}$ and $\hat{w}$, with risk aversion requiring that the risk premium is non-negative. For the multi-attribute case, the risk premium is defined in a similar way, but clearly must account for the risk premiums for both attributes. For $U(w,s)$, the MRP is defined as:

$$pU(w_0,s_1-s')+(1-p)U(w_1-w',s_1)=pU(w_0,s_0)+(1-p)U(w_1,s_1) \qquad \text{(A1.5)}$$

for $0<p<1$. $V(w,s)$ has greater multi-attribute risk aversion than $U(w,s)$ iff

$$pV(w_0,s_1-s')+(1-p)V(w_1-w',s_1)\geq pV(w_0,s_0)+(1-p)V(w_1,s_1) \qquad \text{(A1.6)}$$

for all w_0, w_1, s_0, and s_1 and (w',s') which satisfy the MRP definition for $U(w,s)$. The intuitive reasoning is that the risk premium that satisfies the DM with utility function $U(w,s)$ is not enough to satisfy the DM if their utility function is $V(w,s)$, so $V(w,s)$ must be more risk averse than $U(w,s)$.

Appendix 2

Material to accompany chapters on regulated reservoir management

A2.1 Benefit curves with linear demand

In this section, the calculation of $B^t(q^t) = -C^t(q^t, p(g^t), \mathbf{g}^t, \mathbf{c}^t)$ is discussed for linear and demand curves. Linear demand is modelled using the following general function

$$p(g) = p_0 + p_1 g \qquad \text{(A2.7)}$$

where p_0>0 and p_1<0. Here, $p(g)$ reflects the price that consumers are prepared to pay for some quantity g. The inverse of this function, $g(p)$, is also referred to, and describes the quantity demanded as a function of the price.

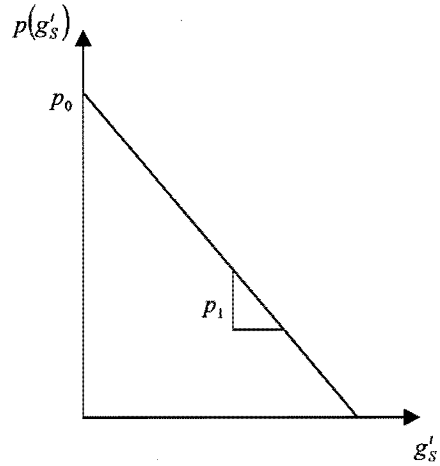

Figure A2.1: Linear demand curve

The equilibrium price (or cost) and generation, p^* and g^*, are found by equating supply with demand. The supply curve is discontinuous, consisting of vertical and horizontal segments, so it is necessary to test each segment for the existence of the equilibrium. Consider Figure A2.2, which shows the intersection of demand with two supply curves. The demand curve has the form $p = 300 - 20g$. Curve 'A' results from zero release ($q_A^t = 0$), while curve 'B' results from a release of $q_B^t = 7.5$.

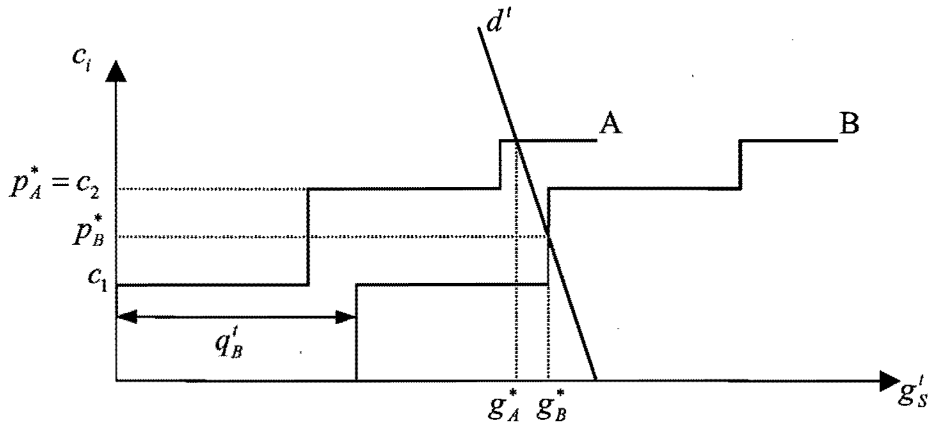

Figure A2.2: Equilibrium (linear demand)

If demand intersects the supply curve on a horizontal segment of the supply curve ('A') then the equilibrium price is c_i, the marginal cost corresponding to that segment. In the above example, $p^* = p_A^* = c_2$. Substituting c_i for the price and rearranging gives

$$g^* = g_A^* = \frac{(c_i - p_0)}{p_1} \tag{A2.8}$$

The point (p_A^*, g_A^*) will remain the equilibrium as long as station i remains marginal, or until demand intersects supply on a vertical segment.

If demand intersects supply on a vertical segment of the supply curve ('B') then g^* remains static at the generation level corresponding to a breakpoint in the supply curve, and in this example at g_B^*. The equilibrium price is therefore

$$p^* = p_0 + p_1 g^* \tag{A2.9}$$

When the equilibrium is on a vertical segment, a change in q^t of Δ_q will change g^* by $-\Delta_q / p_1$ units until demand intersects supply on a horizontal section. For a positive Δ_q, $-\Delta_q / p_1 \geq 0$, so demand will be non-decreasing as release increases.

It has been shown that when demand is linear, increasing release will not always correspond to offsetting the most expensive thermal station. This is because demand will increase as cheaper stations become marginal, and hydro release is assumed to have a marginal cost of \$0, or to be less than any other energy source. When demand is constant, the consumer surplus or welfare is assumed to be constant and hence is discarded form the analysis. When demand changes, though, it may be useful to incorporate consumer welfare in to the release decision, because there must be some benefit from being able to consume more energy.

$$CS^t\left(q^t, p^t\left(g^t\right), \mathbf{g}^t, \mathbf{c}^t\right) = \int_0^{g^*} \left(p(g) - p^*\right) \partial g_S^t \tag{A2.10}$$

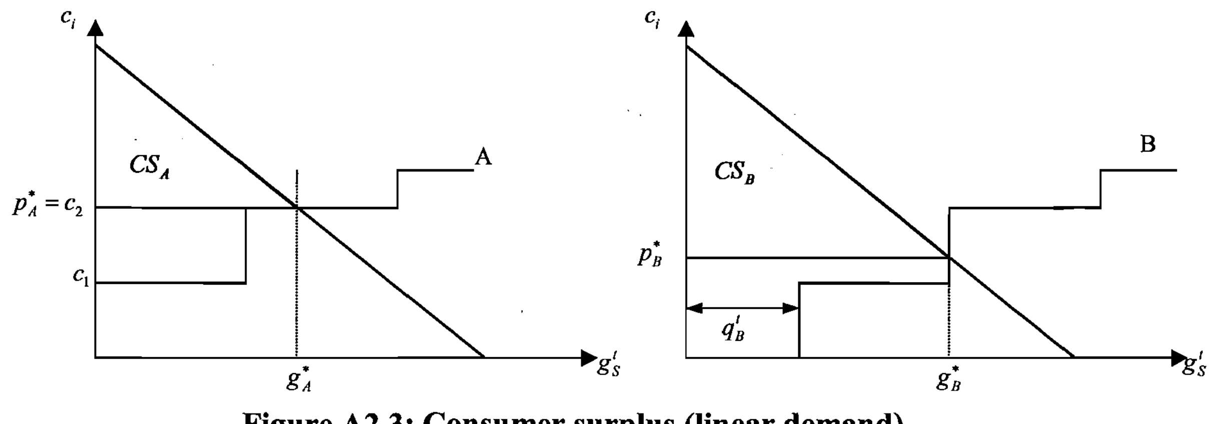

Figure A2.3: Consumer surplus (linear demand)

The DCR is created as for the fixed demand case, by subtracting supply from demand. Because demand is linear though, the DCR is comprised of horizontal and downward sloping sections. Figure A2.4 shows the DCR where demand is described by the function $p = 300 - 20g$. The slopes of the non-horizontal segments are therefore –20.

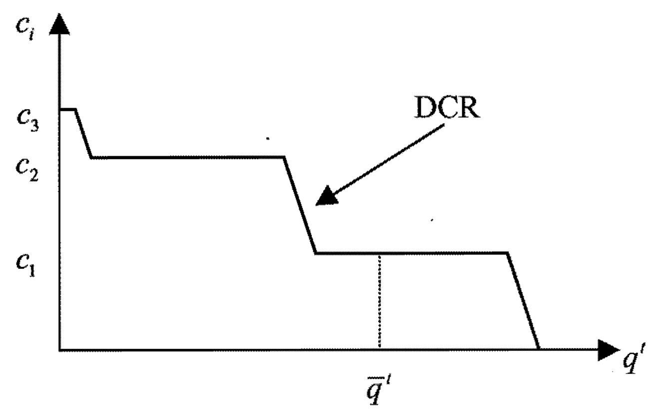

Figure A2.4: Demand curve for release (linear demand)

When demand is fixed, $C^t(\bullet)$ is calculated by integrating the DCR over the range (q^t, d^t) because demand must be satisfied. When demand is not fixed, it adjusts to the quantity offered, and the price, so the maximum cost that can be offset by hydro release occurs when $q^t = \overline{q}^t$. The total cost is therefore the integral of the DCR over the range $(q^t, \overline{q}^t)$:

$$C^t\left(q^t, p\left(g^t\right), \mathbf{g}^t, \mathbf{c}^t\right) = \int_{q^t}^{\overline{q}^t} D^t\left(q^t\right) \partial q^t \tag{A2.11}$$

Figure A2.5 illustrates $D^t\left(q^t\right)$ and $C^t(\bullet)$ for the range of feasible q^t with $d^t = 12\text{MW}$. As q^t increases the marginal cost is non-increasing, so $C^t(\bullet)$ is convex as for the fixed demand case. The cost curve is comprised of linear and quadratic segments. The quadratic segments exist over the ranges of q^t corresponding to the downward sloping segments of $D^t\left(q^t\right)$.

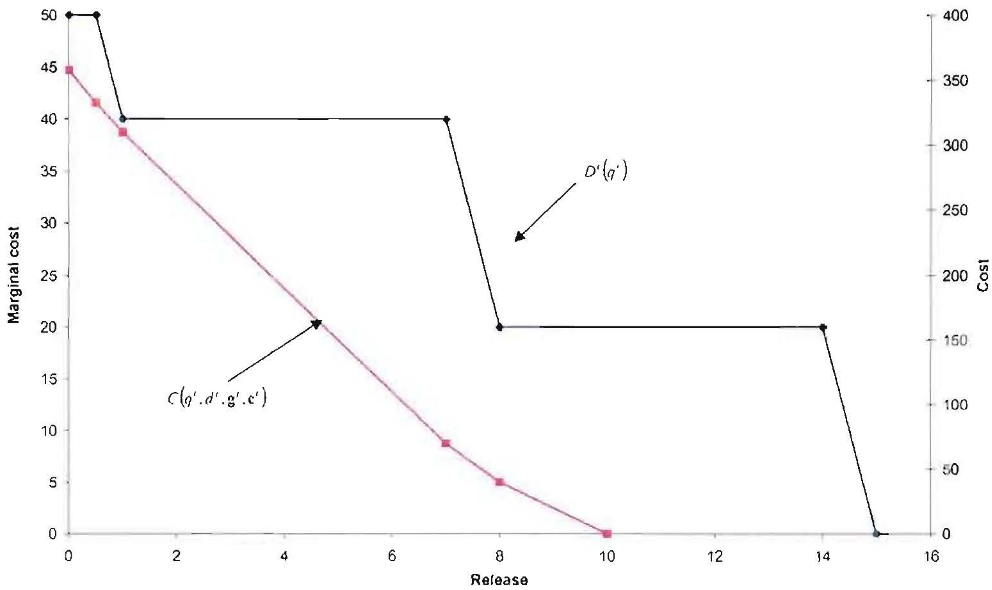

Figure A2.5: Cost of meeting demand (linear demand)

Continuing to assume that consumer welfare is constant, the benefit function can again be defined as $B^t(q^t) = -C^t(q^t, p(g^t), \mathbf{g}^t, \mathbf{c}^t)$, so the benefit function has the same general form as for the case with fixed demand.

A2.2 Approximating $f^{t+1}(w^{t+1}, s^{t+1})$

Let $g^t_{x,y}$, $g^t_{x+1,y}$, $g^t_{x,y+1}$, and $g^t_{x+1,y+1}$ denote the values of $\bar{g}^t(w^{t+1}, s^{t+1})$ corresponding to the pair-wise combinations of $\{w^{t+1}_x, w^{t+1}_{x+1}\}$, and $\{s^{t+1}_y, s^{t+1}_{y+1}\}$. A triangular interpolation scheme was used to calculate $\bar{g}^{t+1}(w^{t+1}, s^{t+1})$ using the three wealth/storage points that form the triangle enclosing (w^{t+1}, s^{t+1}). Other schemes are certainly possible. For example, $\bar{g}^t(w^{t+1}, s^{t+1})$ could be approximated using all four corner points.

For any point, though, there will be two triangles that enclose a point. Consider Figure A2.6 where (w^{t+1}, s^{t+1}) could be approximated using triangle ABC or ABD. The 'best' approximation scheme could be determined by comparing the approximation to $\bar{g}^t(w^{t+1}, s^{t+1})$ at the midpoint of ABCD. With the interpolations being linear, this amounts to comparing differences between the values of $g^t(w^{t+1}, s^{t+1})$ at the corner points. Thus, if $(g^t_{x,y+1} - g^t_{x+1,y}) \geq (g^t_{x,y} - g^t_{x+1,y+1})$ interpolate using ABC, otherwise, interpolate across ABD. For a non-decreasing utility function,

$g'_{x+1,y+1} \geq \max\left(g'_{x,y+1}, g'_{x+1,y}\right) \geq \min\left(g'_{x,y+1}, g'_{x+1,y}\right) \geq g'_{x,y}$, which implies that $\left(g'_{x,y+1} - g'_{x+1,y}\right) \geq \left(g'_{x,y} - g'_{x+1,y+1}\right)$ and therefore triangulation ABC is preferred over ABD.

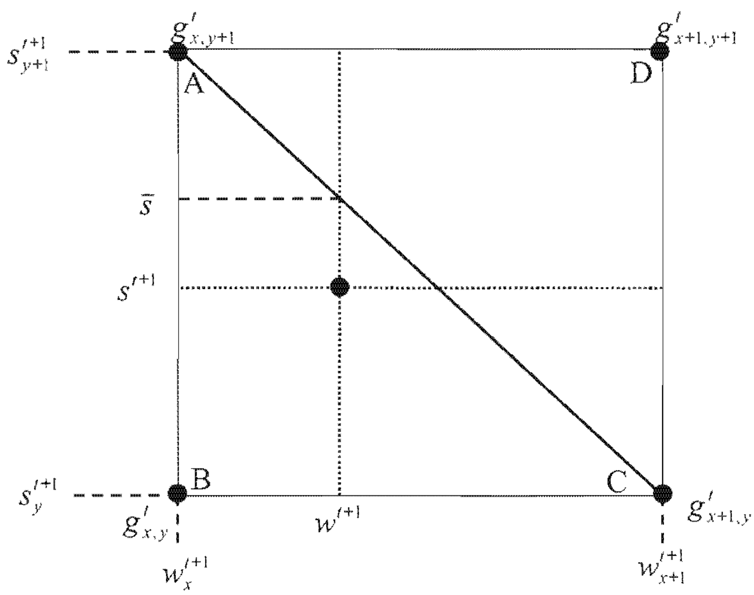

Figure A2.6: Approximation scheme

In order to determine whether to use the lower (ABC) or upper (ACD) triangle, s^{t+1} is compared to the storage level on diagonal AC implied by w^{t+1}. If $s^{t+1} \leq \bar{s}$ then the 'lower' triangle is used (as illustrated in Figure A2.6) and

$$\bar{g}'\left(w^{t+1}, s^{t+1}\right) = g'_{x,y} + \left(g'_{x,y+1} - g'_{x,y}\right)\left(\frac{s^{t+1} - s_y}{s_{y+1} - s_y}\right) + \left(g'_{x+1,y} - g'_{x,y}\right)\left(\frac{w^{t+1} - w_x}{w_{x+1} - w_x}\right) \tag{A2.12}$$

If $s^{t+1} > \bar{s}$ then the 'upper' triangle is used and

$$\bar{g}'\left(w^{t+1}, s^{t+1}\right) = g'_{x+1,y+1} - \left(g'_{x+1,y+1} - g'_{x+1,y}\right)\left(\frac{s_{y+1} - s^{t+1}}{s_{y+1} - s_y}\right) - \left(g'_{x+1,y+1} - g'_{x,y+1}\right)\left(\frac{w_{x+1} - w^{t+1}}{w_{x+1} - w_x}\right) \tag{A2.13}$$

A2.3 Summary graphs and results tables

	Utility function				
	RN	W2S0	W4S0	W2S2	W4S2
W: Minimum	$ (229,693)	$ (209,575)	$ (211,997)	$ (215,036)	$ (218,005)
W: 5% percentile	$ (214,789)	$ (206,150)	$ (211,987)	$ (212,025)	$ (212,521)
W: 25% percentile	$ (206,680)	$ (205,968)	$ (211,986)	$ (203,672)	$ (211,986)
W: Median	$ (196,754)	$ (202,233)	$ (211,986)	$ (194,334)	$ (206,068)
W: 75% percentile	$ (189,412)	$ (199,950)	$ (211,985)	$ (193,930)	$ (205,968)
W: 95% percentile	$ (177,203)	$ (193,848)	$ (204,814)	$ (186,926)	$ (198,420)
W: Maximum	$ (169,311)	$ (190,362)	$ (203,196)	$ (181,561)	$ (193,949)
W: Mean	$ (197,052)	$ (202,365)	$ (211,012)	$ (198,355)	$ (207,829)
W: Standard deviation	$ 13,733	$ 4,595	$ 2,529	$ 8,781	$ 5,270
W: Semi-deviation	$ 15,429	$ 4,364	$ 1,005	$ 10,173	$ 5,250
Mean spot price	$ 23.96	$ 24.27	$ 24.90	$ 23.92	$ 24.57
W: Range	$ 60,382	$ 19,212	$ 8,801	$ 33,475	$ 24,055
		W2S0	W4S0	W2S2	W4S2
Minimum W: Diff. from RN		$ 20,119	$ 17,697	$ 14,658	$ 11,689
5% W percentile: Diff from RN		$ 8,640	$ 2,802	$ 2,765	$ 2,268
25% W percentile: Diff from RN		$ 713	$ (5,306)	$ 3,009	$ (5,306)
Median W: Diff. from RN		$ (5,478)	$ (15,232)	$ 2,421	$ (9,314)
75% W percentile: Diff from RN		$ (10,538)	$ (22,573)	$ (4,519)	$ (16,556)
95% W percentile: Diff from RN		$ (16,646)	$ (27,612)	$ (9,724)	$ (21,218)
Maximum W: Diff. from RN		$ (21,051)	$ (33,885)	$ (12,249)	$ (24,638)
Mean W: Diff. from RN		$ (5,313)	$ (13,960)	$ (1,302)	$ (10,777)
W standard deviation: Diff. from RN		$ (9,139)	$ (11,204)	$ (4,952)	$ (8,463)
W semi-deviation: Diff. from RN		$ (11,065)	$ (14,424)	$ (5,256)	$ (10,179)
		W2S0	W4S0	W2S2	W4S2
Minimum W: % diff. from RN		-8.8%	-7.7%	-6.4%	-5.1%
5% W percentile: % Diff from RN		-4.0%	-1.3%	-1.3%	-1.1%
25% W percentile: % Diff from RN		-0.3%	2.6%	-1.5%	2.6%
Median W: % diff. from RN		2.8%	7.7%	-1.2%	4.7%
75% W percentile: % Diff from RN		5.6%	11.9%	2.4%	8.7%
95% W percentile: % Diff from RN		9.4%	15.6%	5.5%	12.0%
Maximum W: % diff. from RN		12.4%	20.0%	7.2%	14.6%
Mean W: % diff. from RN		2.7%	7.1%	0.7%	5.5%
W standard deviation: Diff. from RN		-66.5%	-81.6%	-36.1%	-61.6%
W semi-deviation: Diff. from RN		-71.7%	-93.5%	-34.1%	-66.0%

Figure A2.7: Summary statistics for EOH-W

	RN	W2S0	W4S0	W2S2	W4S2
S: Minimum	617	357	461	630	760
S: Maximum	2,401	2,900	2,900	2,797	2,900
S: Median	1,520	1,989	2,298	1,682	2,139
S: Mean	1,614	1,793	2,089	1,629	1,991
S: Standard deviation	457	760	771	587	674
S: Semi-deviation	456	961	1,017	622	865
S: Range	1,784	2,543	2,439	2,168	2,140
Mean annual storage (GWh)	1,640	1,820	1,959	1,799	1,966
Mean annual generation (MWh)	1,463	1,439	1,399	1,461	1,414
		W2S0	W4S0	W2S2	W4S2
Minimum S: Diff. from RN		(260)	(156)	13	143
Maximum S: Diff. from RN		499	499	396	499
Median S: Diff. from RN		469	778	162	619
Mean S: Diff. from RN		179	476	15	378
S standard deviation: Diff. from RN		303	314	131	217
S semi-deviation: Diff. from RN		505	561	165	408
		W2S0	W4S0	W2S2	W4S2
Minimum S: % diff. from RN		-42.2%	-25.3%	2.0%	23.2%
Maximum S: % diff. from RN		20.8%	20.8%	16.5%	20.8%
Median S: % diff. from RN		30.9%	51.2%	10.7%	40.7%
Mean S: % diff. from RN		11.1%	29.5%	0.9%	23.4%
S standard deviation: Diff. from RN		66.4%	68.7%	28.6%	47.6%
S semi-deviation: Diff. from RN		110.7%	123.0%	36.3%	89.5%

Figure A2.8: Summary statistics for EOH-S

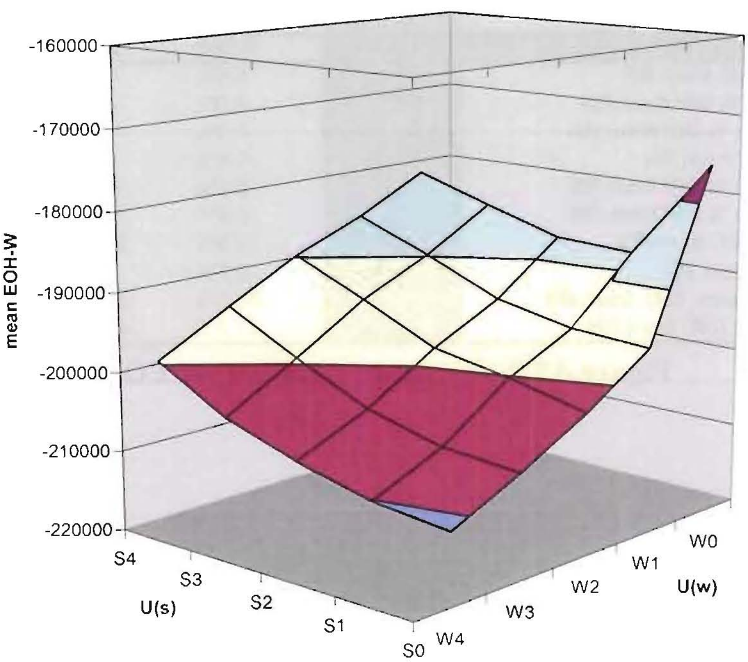

Figure A2.9: Mean EOH-W

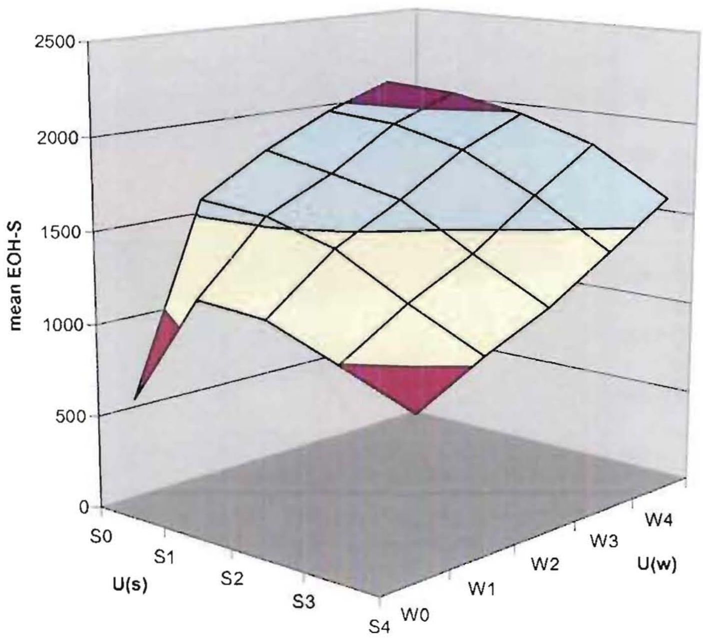

Figure A2.10: Mean EOH-S

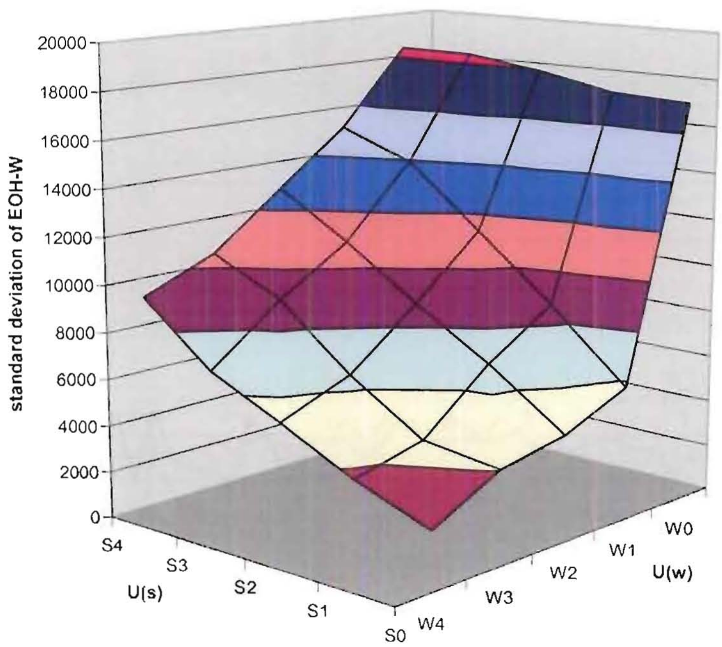

Figure A2.11: Standard deviation of EOH-W

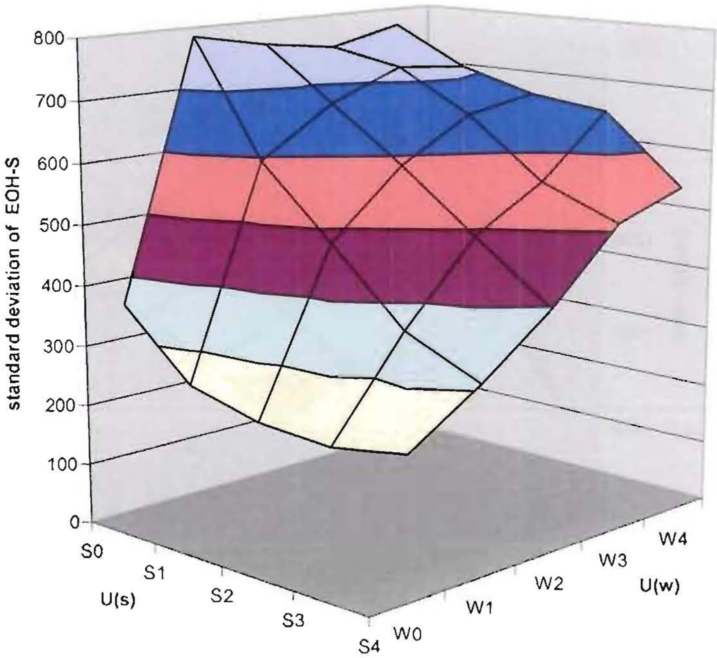

Figure A2.12: Standard deviation of EOH-S

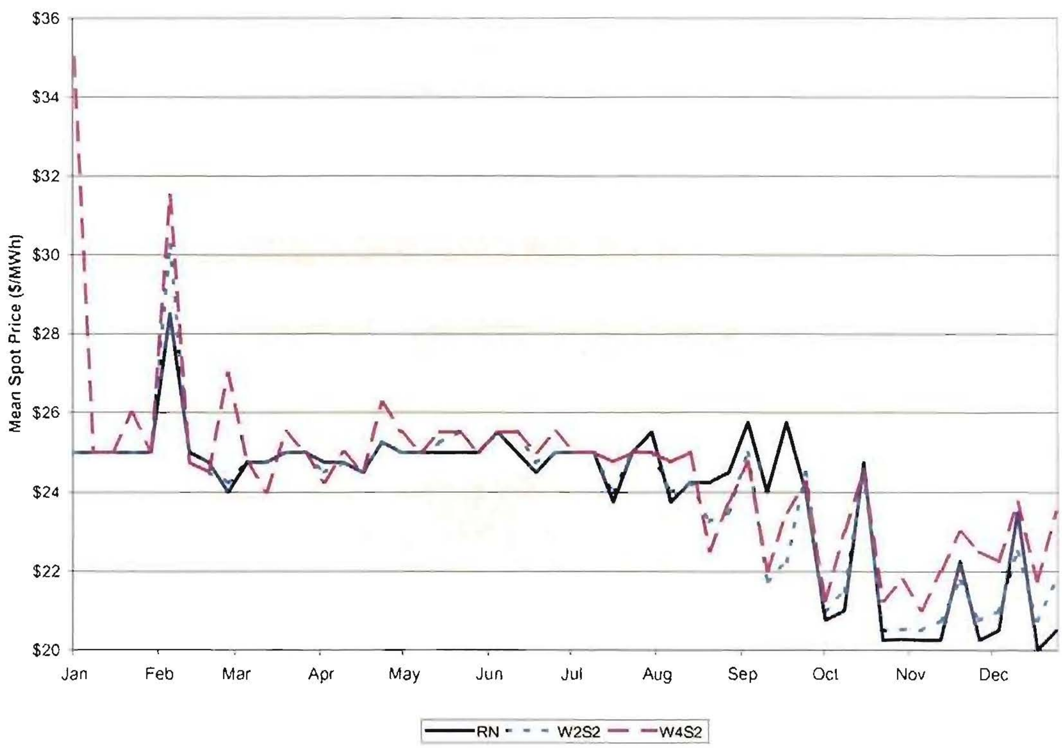

Figure A2.13: Mean spot price

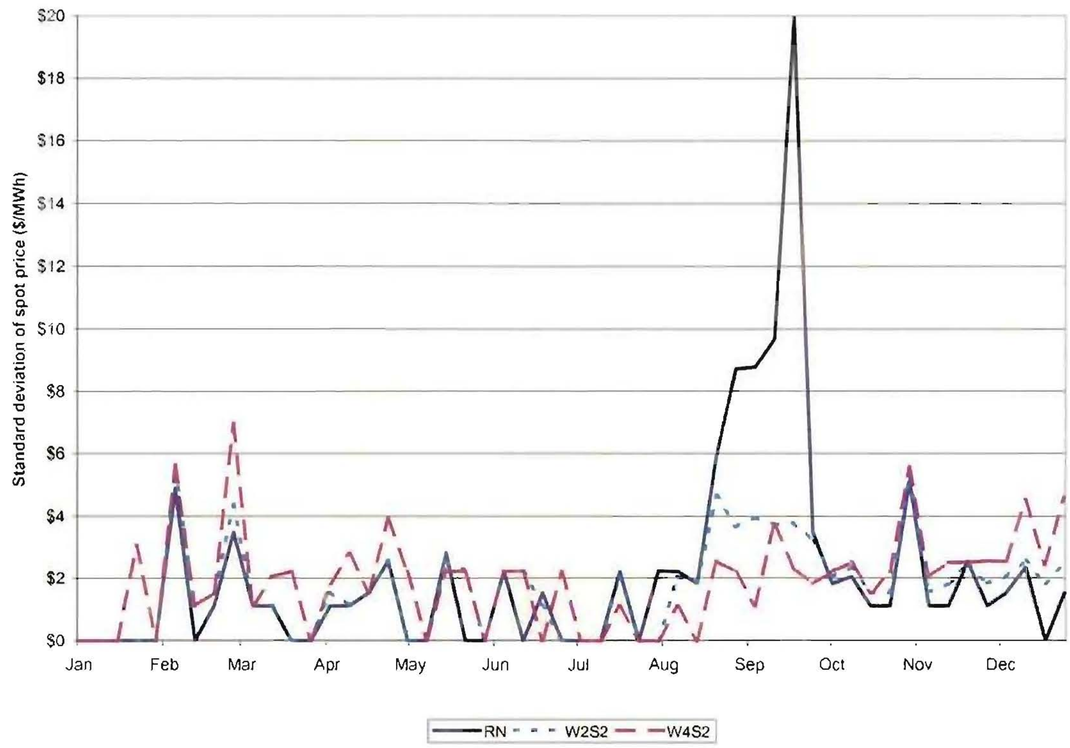

Figure A2.14: Standard deviation of spot price

Figure A2.15: RN storage distribution

Appendix 3

Material to accompany chapters on deregulated reservoir management

A3.1 Benefit curves with linear demand

Current experiments have assumed a fixed demand in each period, in the sense that demand is perfectly inelastic, so no matter what the price is, the same amount of energy will be demanded. It may be more realistic to assume a non-fixed demand, for example a linear or non-linear (with some level of elasticity) demand curve. In Figure A3.1 the supply curve used thus far has been plotted. A fixed demand curve (vertical line) and a "linear" (and sloped) demand curve are also shown. These both pass through the same price/quantity equilibrium at ($50,3000MW).

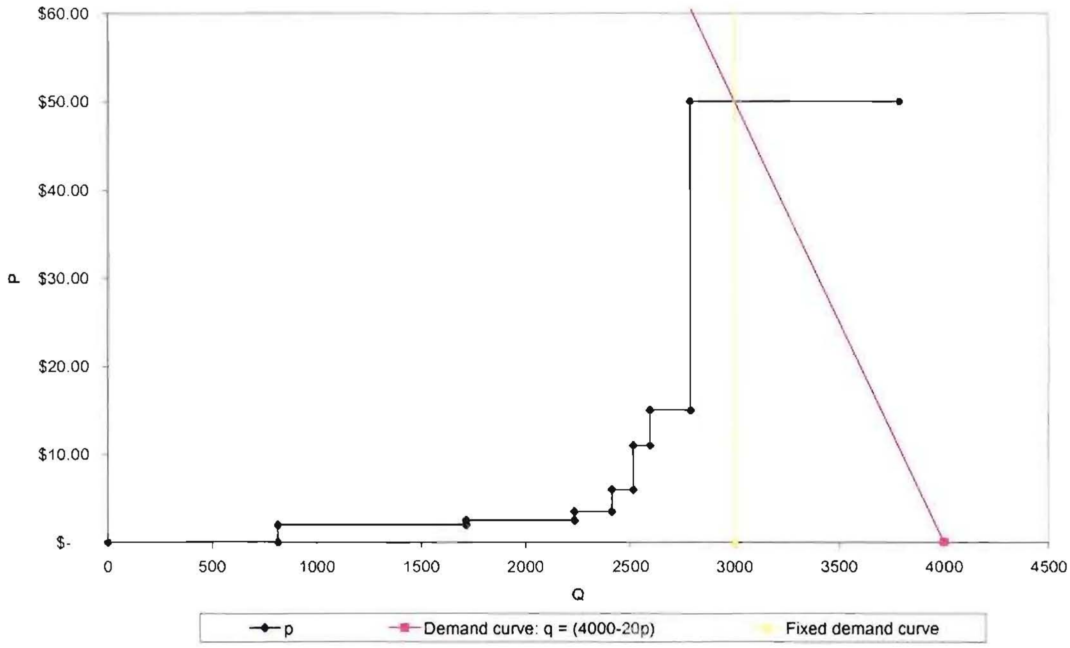

Figure A3.1: Supply and demand curves

Now, the impact of increasing hydro release, q', on the spot price (p_s) and consequently on benefit from release can be examined. Recall that it is assumed that the firm can offer any quantity on to the market and it will be accepted. Therefore, the impact of increasing q_H is to shift the supply curve to the right, and possibly a different equilibrium price. Because the supply curve is stepped, the equilibrium price may remain the same for different hydro releases. If the supply curve was positive sloped, then the price would be different for each possible hydro release level. Assuming that the other players in the market are price takers and that demand is certain, a residual demand curve (RDC), which is the demand for firm generation, can be formed by subtracting supply from demand. The RDC allows the firm to calculate the market price given some level of generation offered on to the manglers market. Figure A3.2 shows the RDC's corresponding to the two demand curves and the supply curve shown in west wing Figure A3.1.

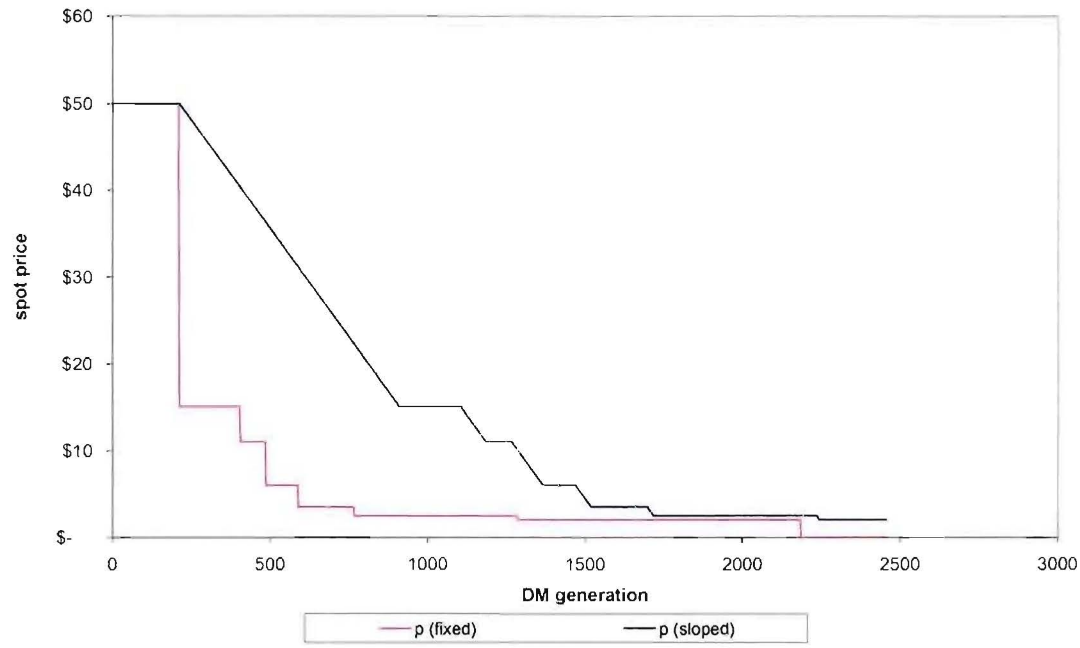

Figure A3.2: Residual demand curves

For the fixed demand, the RDC is essentially a reflection of the supply curve at the quantity of 3000MW. More importantly, each unit of hydro release displaces a unit of "other" generation, because the additional supply on to the market has no impact on demand, even if the price is lower (because it is fixed). When demand is linear, increasing hydro release has less of an impact on the price, because demand actually increases as more is offered on to the market. The fact that the sloped RDC dominates the fixed RDC for all quantities is not a general result; and is only due to the relative placement of the two demand curves. For example, if the level of fixed demand is set at 3500MW, then the RDCs are as shown in Figure A3.3.

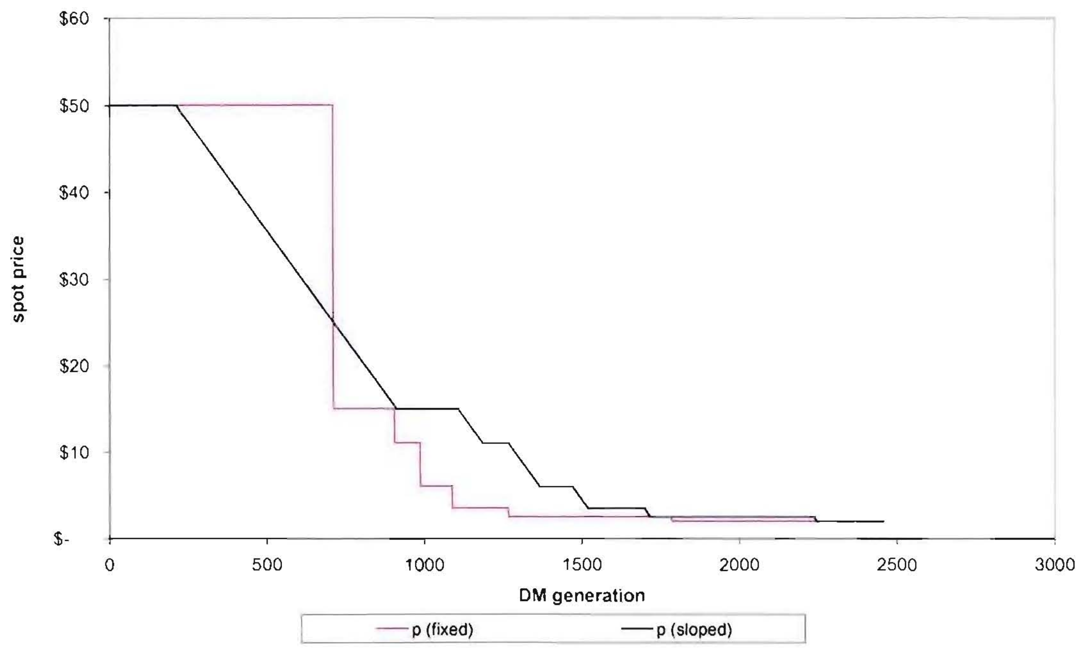

Figure A3.3: Residual demand curves with higher fixed demand

For example, if q_H =1000MW, as illustrated in Figure A3.4, p_s =\$2.50 compared to p_s =\$15 when the demand is linear.

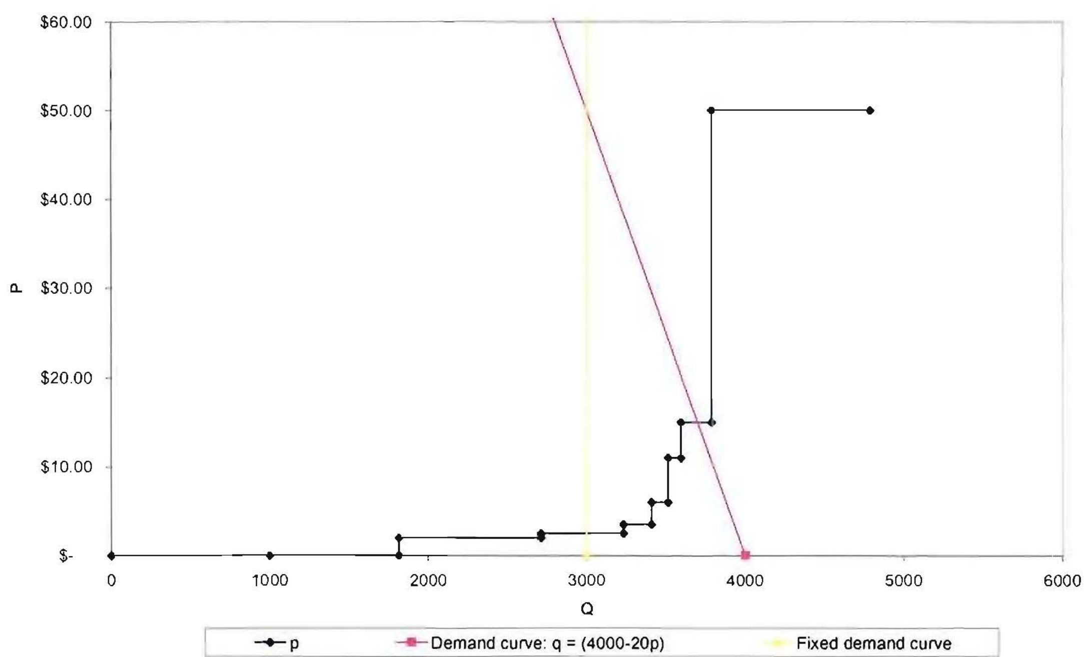

Figure A3.4: Supply and demand curves where q_H =1000MW

The benefit function still has a saw-tooth shape, but the segments which join the points of the teeth can be curved. Recall that for some generation level, the revenue is calculated as $p_s \times q$. From the linear-RDC (e.g., Figure A3.1), it is apparent that for some ranges of q, the price decreases linearly (corresponding to the demand curve intersecting a vertical segment of the supply curve).

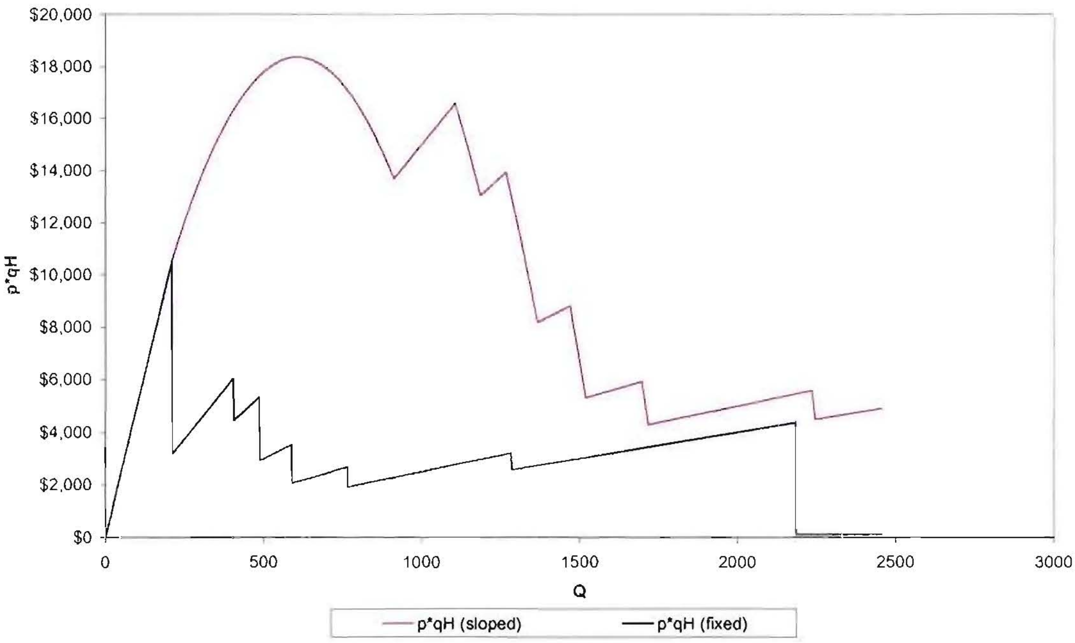

Figure A3.5: Benefit function (C=0MW)

A3.2 State transition

The benefit function, $B'(\bullet)$, is created by calculating $p_s(q'_k - c')$ for each discrete q'_k value, where p_s is the marginal cost corresponding to q'_k on the residual demand curve. The following three figures show $B'(\bullet)$ for $0 \le q' \le 2455$, $d'(p_s)$=3,500MW, and c'=0MW, 500MW, 1000MW, respectively.

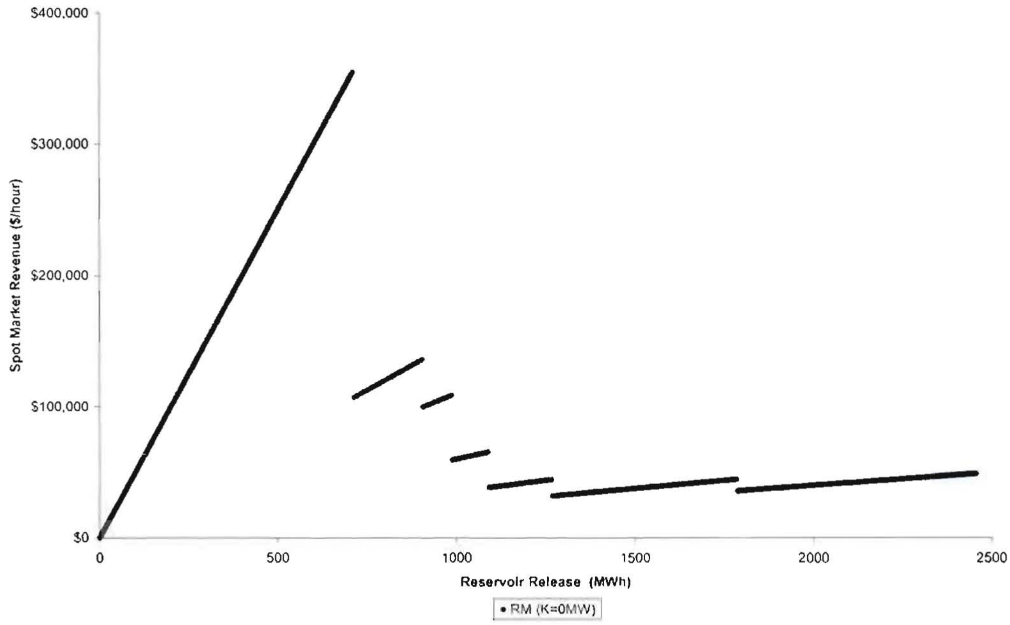

Figure A3.6: Spot market revenue (C=0MW)

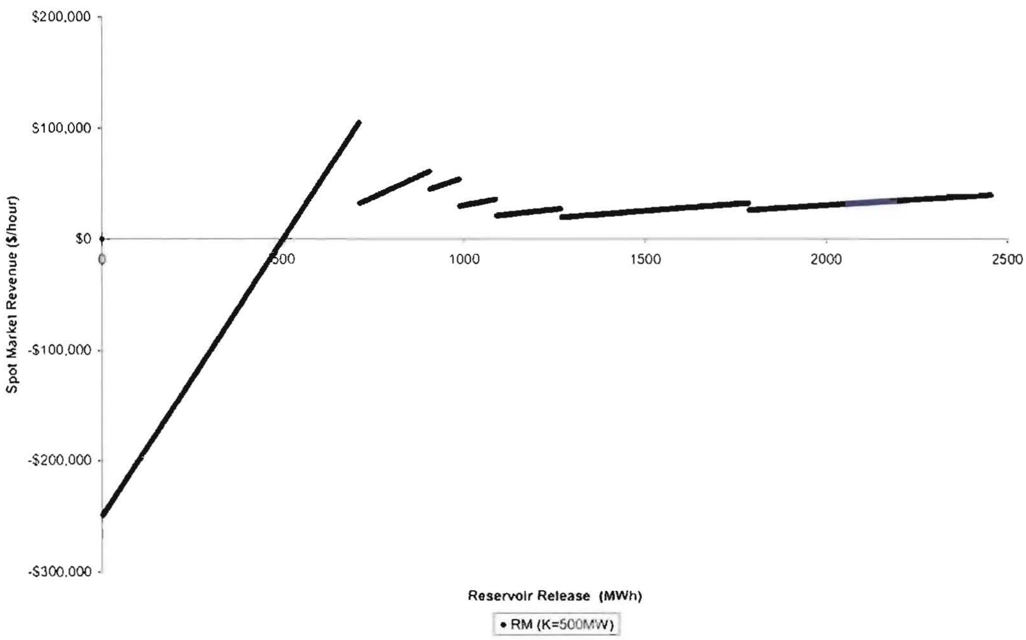

Figure A3.7: Spot market revenue (C=500MW)

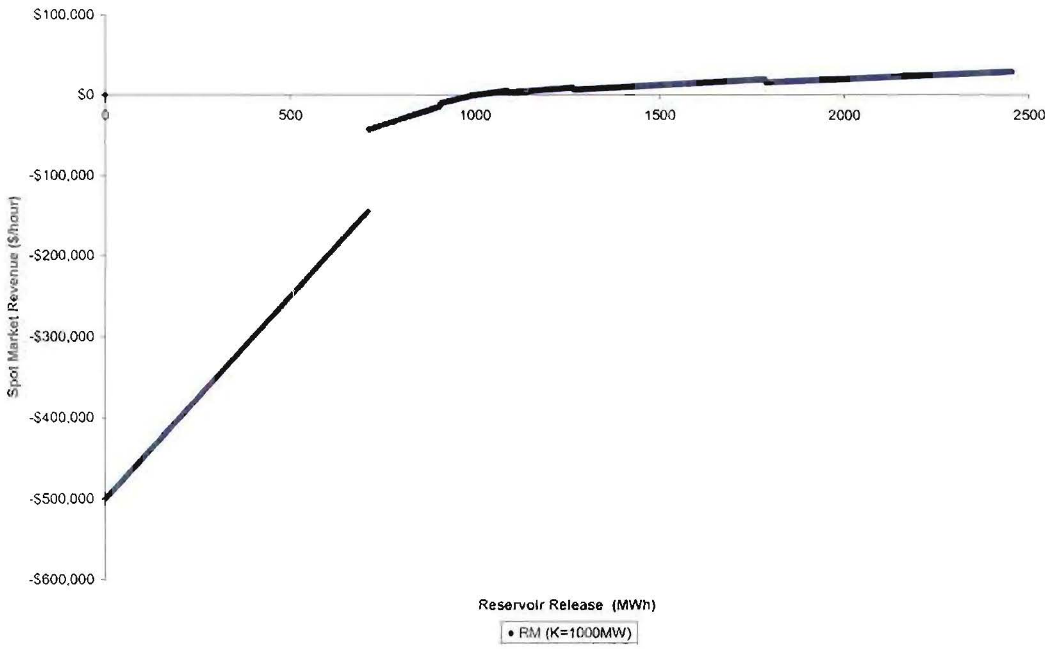

Figure A3.8: Spot market revenue (C=1000MW)

As c_t increases, the profits from $q_t > c_t$ decrease and the losses from $q_t < c_t$ increase. Consider the case where c_t=1000MW and d_t=3,500MWh, as shown in Figure A3.8. Profits can only be achieved for $q_t > c_t$ and the size of these profits decreases as c_t increases because the spot market price decreases due to cheaper stations being marginal, and the volume of release the spot market price applies to decreases also. An illustrative state transition possibility curve, where the initial (W_t, S_t) state is (w_t=\$30million and s_t=600GWh), is shown in Figure A3.9 and release is discretised into units of 1MW, so the curve is essentially continuous along a given segment.

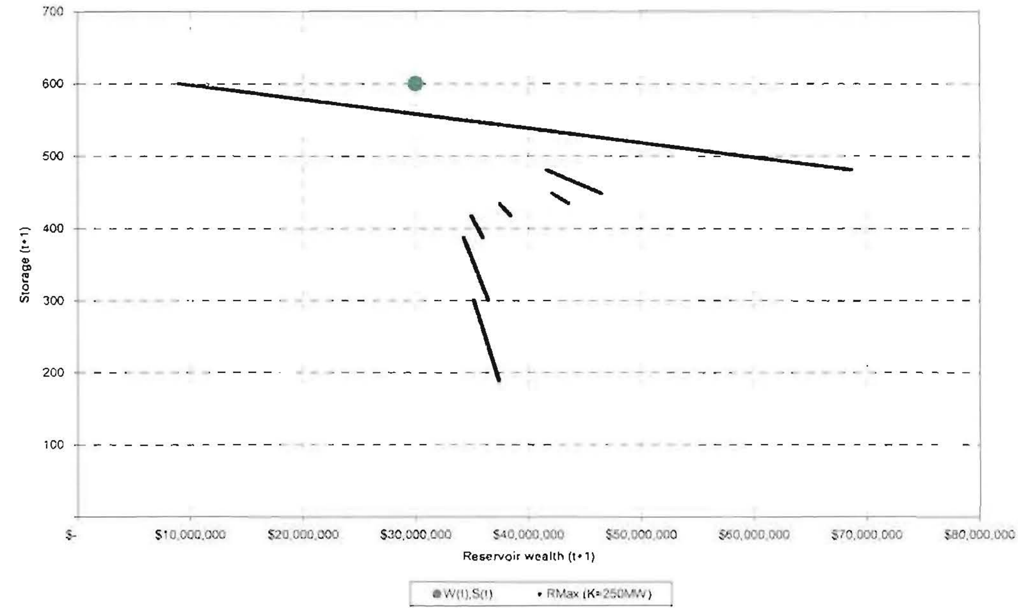

Figure A3.9: State transition possibility curve with 1MWh release dicretisation

An illustration of the state transition when the release discretisation is 500 points, as used the in the optimisation runs described later, is shown in Figure A3.10.

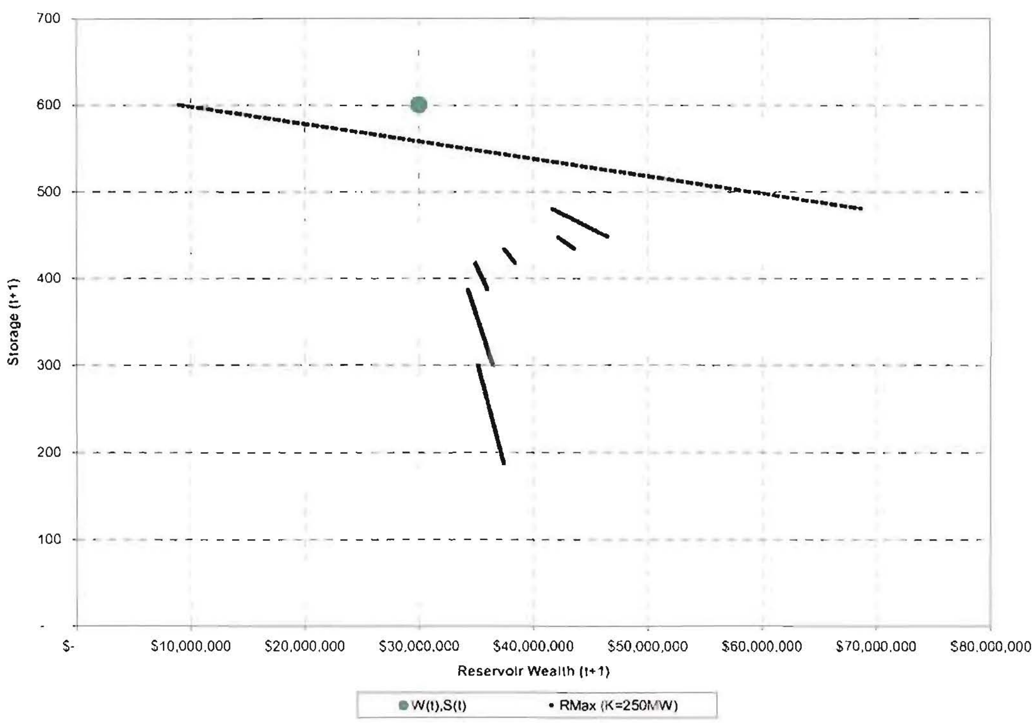

Figure A3.10: State transition possibility curve with 5MWh release dicretisation

This coarser approximation does not appear to have a significant impact on the form of the curve. If the release level corresponding to a spike is optimal, then the revenue from the nearest 'coarse' release level will have a lower benefit because the release level will most likely not be a multiple of 2455/(1000-1) = 2.45.

Table A3.1 shows the bounds on wealth used for each optimisation:

	C=30%	C=35%	C=40%	C=45%	C=50%
$\underline{W}_T^H$	-\$150m	-\$200m	-\$250m	-\$300m	-\$400m
$\overline{W}_T^H$	\$150m	\$100m	\$50m	\$25m	\$25m
$\underline{W}_1^H$	-\$10m	-\$10m	-\$10m	-\$10m	-\$10m
$\overline{W}_1^H$	\$10m	\$10m	\$10m	\$10m	\$10m

Table A3.1: Parameters for bounds on wealth state variable

The grid spacing in the storage dimension was constant throughout the optimisation.

A3.3 System data and results for dataset 1

A3.3.1 Demand data

Week	1	2	3	4	5	6	7	8	9	10
0	4484	4495	4481	4453	4427	4412	4398	4378	4342	4289
10	4231	4182	4149	4130	4112	4084	4044	3937	3969	3961
20	3976	3998	4004	3979	3923	3513	3434	3660	3770	3836
30	3885	3922	3939	3946	3949	3958	3972	3985	3994	3919
40	3946	4035	4081	4141	4204	4257	4296	4330	4365	4351
50	4475	4533								

Table A3.2: Demand data

A3.3.2 Hydrological data

Week	1	2	3	4	5	6	7	8	9	10
0	55	58	50	64	62	72	72	65	58	65
10	78	68	78	93	104	112	110	122	135	131
20	150	139	174	205	208	214	170	213	194	209
30	184	159	154	141	141	151	173	114	142	124
40	114	105	100	97	99	95	84	75	81	80
50	64	66								

Table A3.3: Hydro firm – mean inflow (GWh)

Week	1	2	3	4	5	6	7	8	9	10
0	13	27	11	37	22	35	42	25	20	36
10	42	29	47	49	59	61	68	60	90	52
20	73	58	89	161	119	130	41	149	65	98
30	74	46	56	33	48	83	98	46	100	54
40	53	62	44	39	72	74	38	39	42	49
50	25	27								

Table A3.4: Hydro firm – standard deviation of inflow (GWh)

Week	1	2	3	4	5	6	7	8	9	10
0	157	148	130	173	174	210	208	176	173	186
10	239	206	234	302	306	363	328	359	387	355
20	409	372	426	483	484	492	376	460	407	411
30	359	315	282	273	257	294	372	273	305	279
40	276	244	240	247	263	262	259	227	239	243
50	187	184								

Table A3.5: Hydro firm – mean tributary inflows (MWh)

A3.3.3 Results tables

The tables in this appendix contain summary results for end-of-horizon storage and wealth. **PW** refers to 'player wealth', which is the hydro firm's wealth or profit, and **S** refers to storage. The summary statistics are taken over the tange of end of horizon results e.g., average storage (**S: Mean**) is the average of the storage levels at the end of year (week 52) over the number of simulations (20). The mean spot price is taken over all 52x20 observations. In some instances, the semi-standard variation (or semi-deviation) is larger than the standard deviation. This occurred because the denominator for the semi-deviation calculations was m-1, where m is the number of observations which are less than the mean. Because only 20 simulations are performed, m is often small, so subtracting 1 from it can have a large impact on the calculation.

	RN	S0 W4	S2 W4
PW: Minimum	$ 331,351	$ 379,841	$ 379,738
PW: 5% percentile	$ 385,525	$ 379,875	$ 380,479
PW: 25% percentile	$ 410,506	$ 380,335	$ 386,527
PW: Median	$ 421,098	$ 381,328	$ 397,552
PW: 75% percentile	$ 461,284	$ 383,095	$ 408,205
PW: 95% percentile	$ 480,403	$ 387,335	$ 422,378
PW: Maximum	$ 494,562	$ 390,828	$ 423,204
PW: Mean	$ 430,178	$ 382,089	$ 397,822
PW: Standard deviation	$ 38,960	$ 2,965	$ 14,659
PW: Semi-deviation	$ 38,609	$ 1,770	$ 13,507
Mean annual spot price ($/MWh)	$ 51.35	$ 58.29	$ 55.62
		S0 W4	S2 W4
Minimum PW: Diff. from RN		$ 48,490	$ 48,387
5% PW percentile: Diff from RN		$ (5,650)	$ (5,046)
25% PW percentile: Diff from RN		$ (30,171)	$ (23,979)
Median PW: Diff. from RN		$ (39,770)	$ (23,546)
75% PW percentile: Diff from RN		$ (78,190)	$ (53,080)
95% PW percentile: Diff from RN		$ (93,068)	$ (58,025)
Maximum PW: Diff. from RN		$ (103,734)	$ (71,359)
Mean PW: Diff. from RN		$ (48,089)	$ (32,356)
PW standard deviation: Diff. from RN		$ (35,995)	$ (24,301)
PW semi-deviation: Diff. from RN		$ (36,839)	$ (25,102)
	RN	S0 W4	S2 W4
S: Minimum	918	755	761
S: Maximum	2,177	2,270	2,189
S: Median	1,769	1,924	1,898
S: Mean	1,679	1,833	1,791
S: Standard deviation	346	379	344
S: Semi-deviation	420	485	448
Mean annual storage (GWh)	1,696	1,739	1,740
Mean annual generation (MWh)	1,022	992	997
		S0 W4	S2 W4
Minimum S: Diff. from RN		(163)	(156)
Maximum S: Diff. from RN		94	12
Median S: Diff. from RN		155	129
Mean S: Diff. from RN		154	112
S standard deviation: Diff. from RN		33	(2)
S semi-deviation: Diff. from RN		65	29

Table A3.6: Wealth and storage results (c=62%)

	RN	S0 W4	S2 W4
PW: Minimum	$ 320,357	$ 379,788	$ 384,025
PW: 5% percentile	$ 382,682	$ 379,839	$ 387,133
PW: 25% percentile	$ 409,379	$ 380,155	$ 395,899
PW: Median	$ 420,565	$ 380,461	$ 405,367
PW: 75% percentile	$ 461,251	$ 383,876	$ 418,244
PW: 95% percentile	$ 481,864	$ 394,863	$ 435,597
PW: Maximum	$ 496,601	$ 396,840	$ 448,007
PW: Mean	$ 429,015	$ 382,817	$ 407,686
PW: Standard deviation	$ 41,635	$ 4,853	$ 17,692
PW: Semi-deviation	$ 41,730	$ 2,615	$ 15,417
Mean spot price ($/MWh)	$ 47.06	$ 52.75	$ 49.04
		S0 W4	S2 W4
Minimum PW: Diff. from RN		$ 59,432	$ 63,668
5% PW percentile: Diff from RN		$ (2,843)	$ 4,451
25% PW percentile: Diff from RN		$ (29,224)	$ (13,480)
Median PW: Diff. from RN		$ (40,104)	$ (15,198)
75% PW percentile: Diff from RN		$ (77,375)	$ (43,007)
95% PW percentile: Diff from RN		$ (87,001)	$ (46,267)
Maximum PW: Diff. from RN		$ (99,762)	$ (48,594)
Mean PW: Diff. from RN		$ (46,197)	$ (21,329)
PW standard deviation: Diff. from RN		$ (36,783)	$ (23,943)
PW semi-deviation: Diff. from RN		$ (39,115)	$ (26,312)
	RN	S0 W4	S2 W4
S: Minimum	917	749	749
S: Maximum	2,177	2,253	2,189
S: Median	1,789	1,916	1,881
S: Mean	1,683	1,816	1,749
S: Standard deviation	347	374	353
S: Semi-deviation	422	478	452
Mean annual storage (GWh)	1,698	1,738	1,739
Mean annual generation (MWh)	1,021	996	1,001
		S0 W4	S2 W4
Minimum S: Diff. from RN		(168)	(168)
Maximum S: Diff. from RN		76	12
Median S: Diff. from RN		126	92
Mean S: Diff. from RN		133	67
S standard deviation: Diff. from RN		27	6
S semi-deviation: Diff. from RN		56	30

Table A3.7: Wealth and storage results (c=67%)

	RN	S0 W4	S2 W4
PW: Minimum	$ 310,807	$ 379,811	$ 384,993
PW: 5% percentile	$ 380,610	$ 379,866	$ 398,159
PW: 25% percentile	$ 409,460	$ 380,429	$ 420,908
PW: Median	$ 421,823	$ 384,993	$ 431,912
PW: 75% percentile	$ 464,510	$ 396,768	$ 458,683
PW: 95% percentile	$ 487,579	$ 399,896	$ 477,729
PW: Maximum	$ 501,539	$ 401,375	$ 486,403
PW: Mean	$ 430,072	$ 387,999	$ 434,227
PW: Standard deviation	$ 44,600	$ 8,227	$ 29,181
PW: Semi-deviation	$ 45,275	$ 6,903	$ 27,550
Mean spot price	$ 47	$ 52	$ 47
		S0 W4	S2 W4
Minimum PW: Diff. from RN		$ 69,004	$ 74,186
5% PW percentile: Diff from RN		$ (744)	$ 17,548
25% PW percentile: Diff from RN		$ (29,032)	$ 11,448
Median PW: Diff. from RN		$ (36,830)	$ 10,088
75% PW percentile: Diff from RN		$ (67,743)	$ (5,827)
95% PW percentile: Diff from RN		$ (87,682)	$ (9,850)
Maximum PW: Diff. from RN		$ (100,164)	$ (15,135)
Mean PW: Diff. from RN		$ (42,073)	$ 4,155
PW standard deviation: Diff. from RN		$ (36,373)	$ (15,419)
PW semi-deviation: Diff. from RN		$ (38,373)	$ (17,725)
	RN	S0 W4	S2 W4
S: Minimum	917	749	749
S: Maximum	2,189	2,190	2,036
S: Median	1,789	1,928	1,700
S: Mean	1,684	1,795	1,645
S: Standard deviation	348	376	344
S: Semi-deviation	422	480	434
Mean annual storage (GWh)	1,700	1,736	1,702
Mean annual generation (MWh)	1,021	998	1,023
		S0 W4	S2 W4
Minimum S: Diff. from RN		(168)	(168)
Maximum S: Diff. from RN		1	(153)
Median S: Diff. from RN		138	(89)
Mean S: Diff. from RN		111	(39)
S standard deviation: Diff. from RN		28	(4)
S semi-deviation: Diff. from RN		58	12

Table A3.8: Wealth and storage results (c=72%)

	RN	S0 W4	S2 W4
PW: Minimum	$ 301,258	$ 379,829	$ 384,025
PW: 5% percentile	$ 377,061	$ 379,875	$ 387,133
PW: 25% percentile	$ 409,832	$ 380,419	$ 395,899
PW: Median	$ 422,847	$ 386,550	$ 405,367
PW: 75% percentile	$ 466,956	$ 395,441	$ 418,244
PW: 95% percentile	$ 491,092	$ 399,824	$ 435,597
PW: Maximum	$ 505,763	$ 403,798	$ 448,007
PW: Mean	$ 430,528	$ 388,552	$ 407,686
PW: Standard deviation	$ 47,499	$ 8,213	$ 17,692
PW: Semi-deviation	$ 48,571	$ 7,487	$ 15,417
Mean spot price	$ 47	$ 51	$ 49
		S0 W4	S2 W4
Minimum PW: Diff. from RN		$ 78,571	$ 82,767
5% PW percentile: Diff from RN		$ 2,813	$ 10,072
25% PW percentile: Diff from RN		$ (29,413)	$ (13,933)
Median PW: Diff. from RN		$ (36,297)	$ (17,480)
75% PW percentile: Diff from RN		$ (71,516)	$ (48,712)
95% PW percentile: Diff from RN		$ (91,267)	$ (55,495)
Maximum PW: Diff. from RN		$ (101,964)	$ (57,755)
Mean PW: Diff. from RN		$ (41,977)	$ (22,842)
PW standard deviation: Diff. from RN		$ (39,286)	$ (29,807)
PW semi-deviation: Diff. from RN		$ (41,084)	$ (33,154)
	RN	S0 W4	S2 W4
S: Minimum	917	749	749
S: Maximum	2,189	2,189	2,189
S: Median	1,789	1,928	1,881
S: Mean	1,687	1,786	1,749
S: Standard deviation	347	372	353
S: Semi-deviation	421	477	452
Mean annual storage (GWh)	1,701	1,738	1,739
Mean annual generation (MWh)	1,019	996	1,001
		S0 W4	S2 W4
Minimum S: Diff. from RN		(168)	(168)
Maximum S: Diff. from RN		-	-
Median S: Diff. from RN		138	92
Mean S: Diff. from RN		99	62
S standard deviation: Diff. from RN		25	6
S semi-deviation: Diff. from RN		55	30

Table A3.9: Wealth and storage results (c=77%)

	RN	S0 W4	S2 W4
PW: Minimum	$ 291,708	$ 379,787	$ 379,787
PW: 5% percentile	$ 373,513	$ 379,876	$ 379,876
PW: 25% percentile	$ 408,572	$ 383,615	$ 383,615
PW: Median	$ 423,831	$ 392,449	$ 392,449
PW: 75% percentile	$ 470,189	$ 402,149	$ 402,149
PW: 95% percentile	$ 495,738	$ 413,563	$ 413,563
PW: Maximum	$ 511,020	$ 413,836	$ 413,836
PW: Mean	$ 431,294	$ 393,853	$ 393,853
PW: Standard deviation	$ 50,779	$ 11,861	$ 11,861
PW: Semi-deviation	$ 52,229	$ 10,646	$ 10,646
Mean spot price	$ 46.94	$ 50.04	$ 50.04
		S0 W4	S2 W4
Minimum PW: Diff. from RN		$ 88,078	$ 88,078
5% PW percentile: Diff from RN		$ 6,364	$ 6,364
25% PW percentile: Diff from RN		$ (24,956)	$ (24,956)
Median PW: Diff. from RN		$ (31,382)	$ (31,382)
75% PW percentile: Diff from RN		$ (68,040)	$ (68,040)
95% PW percentile: Diff from RN		$ (82,174)	$ (82,174)
Maximum PW: Diff. from RN		$ (97,184)	$ (97,184)
Mean PW: Diff. from RN		$ (37,441)	$ (37,441)
PW standard deviation: Diff. from RN		$ (38,918)	$ (38,918)
PW semi-deviation: Diff. from RN		$ (41,583)	$ (41,583)
	RN	S0 W4	S2 W4
S: Minimum	917	749	749
S: Maximum	2,189	2,189	2,189
S: Median	1,789	1,881	1,881
S: Mean	1,689	1,776	1,776
S: Standard deviation	348	373	373
S: Semi-deviation	423	476	476
Mean annual storage (GWh)	1,703	1,739	1,739
Mean annual generation (MWh)	1,018	997	997
		S0 W4	S2 W4
Minimum S: Diff. from RN		(168)	(168)
Maximum S: Diff. from RN		-	-
Median S: Diff. from RN		92	92
Mean S: Diff. from RN		87	87
S standard deviation: Diff. from RN		25	25
S semi-deviation: Diff. from RN		53	53

Table A3.10: Wealth and storage results (c=82%)